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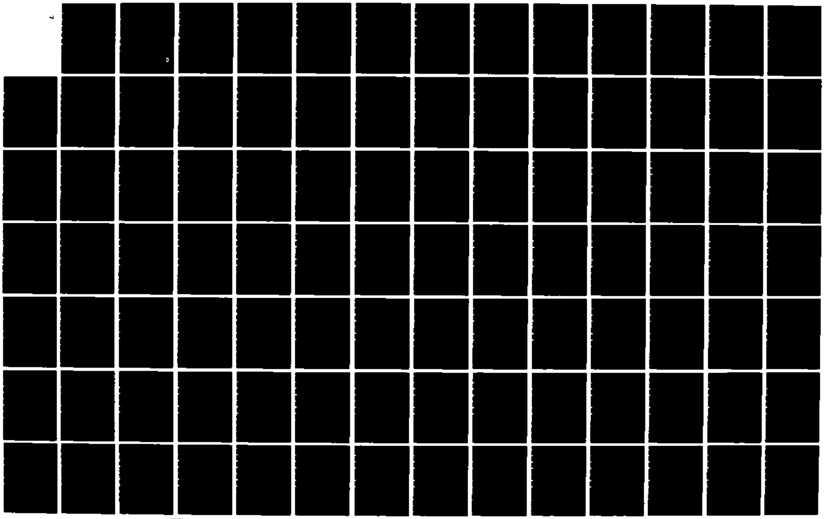

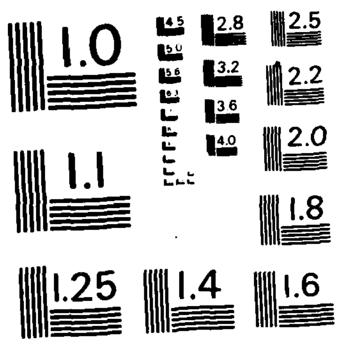

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DATA ENVELOPMENT ANALYSIS AND EXTENSIONS FOR

DECISION SUPPORT AND MANAGEMENT PLANNING

by

CHARLES TERRANCE CLARK, B.A., M.S.

DISSERTATION

Presented to the Faculty of the Graduate School of

The University of Texas at Austin

in Partial Fulfillment

of the Requirements

for the Degree of

DOCTOR OF PHILOSOPHY

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May, 1983

DATA ENVELOPMENT ANALYSIS AND EXTENSIONS FOR

DECISION SUPPORT AND MANAGEMENT PLANNING

APPROVED BY SUPERVISORY COMMITTEE:

Authella Bessent

Darwin Klingman

Joyce Elam

James R. McLean

Richard L Olson

DEDICATION

To my loving wife, Jane,

and my delightful daughters,

LeeAnn and Stephanie, who

were sources of strength,

motivation and comfort

throughout this difficult endeavor

A C K N O W L E D G M E N T S

I am deeply grateful for the encouragement and support I received from Air Force friends, faculty members and my family during the preparation of this study.

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members provided valuable advice, assistance and constructive criticism during the course of this study. Although not an official member of my committee, Dr. Wailand Bessent must also be thanked for the numerous hours he spent advising and helping me.

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Finally, I owe my greatest debt to my wife Jane who consoled me during difficult times, understood my moods and distractions, typed manuscripts for me, assumed extra duties at home, and in general provided the motiva- and encouragement I needed to complete this study.

C. T. C.

April 11, 1983

DATA ENVELOPMENT ANALYSIS AND EXTENSIONS FOR DECISION SUPPORT AND MANAGEMENT PLANNING

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Supervising Professor: Authella Bessent

The public expects military efficiency from the combat forces it supports with tax dollars. The United States Air Force needs integrative measures of efficiency and needs decision support systems which aid in detecting inefficiencies, diagnosing problems, and choosing among alternative courses of action to improve the efficiency and effectiveness of combat units. The Data Envelopment Analysis (DEA) technique developed by Charnes, Cooper and Rhodes provided the basic theoretical starting point for this dissertation. It enables the unified analysis of multiple technical, economic and effectiveness measures in contrast to past reliance by Air Force Management on "partial" measures of productivity, cost effectiveness, etc.

Theory was extended by this study to provide analytical capabilities suitable for use by the Air Force in the analysis and interpretation of efficiency and in

the preparation of management plans. These extensions include methods for post DEA analysis to detect rates of substitution and marginal productivities in nearby frontier facets, facets which if possible are formed solely from empirically observed values. Such methods are needed in developing resource allocation models and in establishing realistic output expectations in management plans. Two important managerial questions related to this research are: (1) how should resources be allocated or technologies be changed to improve the collective efficiency and effectiveness of units? and (2) how can the performance of efficient units be used to predict the expected output levels associated with various input combinations?

The final step was to evaluate the suitability of DEA and the aforementioned extensions in measuring and evaluating the comparative efficiency of a hypothetical set of Air Force units using realistic, insightful data. This included selecting relevant measures of input and output and then performing a trial analysis.

This dissertation provided a basic theoretical framework for future development of decision support prototypes suitable for use by the Air Force in managing military effectiveness and efficiency.

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CHAPTER I

THE PROBLEMS OF EVALUATING AND MANAGING THE CAPABILITY AND EFFICIENCY OF AIR FORCE UNITS

> Waste and inefficiency never fed a hungry child, provided a job for a willing worker, or educated a deserving student.
>
> - Former President Carter [12]

Introduction

A. Charnes, W. W. Cooper, A. Bessent, and W. Bessent (all of The University of Texas at Austin), together with E. Rhodes (State University of New York) have developed and tested a new method called DEA (Data Envelopment Analysis) for measuring and evaluating the efficiency of not-for-profit enterprises in a variety of contexts [6, 18, 19]. The latter include applications to both military and civilian problems which have all involved multiple-output as well as multiple-input situations for each of a variety of managerial Decision Making Units--hereafter called DMU's--where a measure of efficiency was desired which would not require *a priori* weights or similar devices to arrive at a single overall (scalar) measure of efficiency for each such DMU. Furthermore, the models from which this measure is derived also provide details on

the sources and relative magnitudes of any inefficiencies along with information on trade offs and other information needed for improved decision making.

DEA, with a few modifications, appears to have significant potential for near term use by the military in assessing and managing the many facets of efficiency and capability. It enables the unified analysis of multiple technical, economic and effectiveness measures in contrast to past management and analysis techniques which relied too heavily on "partial" measures of productivity, cost effectiveness, maintainability, etc.

Air Force commanders and resource managers need a tool for monitoring the efficiency of combat units, which simultaneously takes into account many of the factors including mix and other variations that might affect combat potential. The measure to be used for these purposes should be theoretically and logically justified in its ability to evaluate the actual achievement of each unit relative to the maximum achieved by other comparable units. The measure should be fair and take into account controllable and uncontrollable variables. It should, on the one hand, provide a convenient summary in the form of a single measure of efficiency and, on the other hand, make it

possible to detect inefficiencies and direct attention to the relevant factors for correcting these inefficiencies. It should further reveal possible trade offs between different inputs and outputs, even when a wing is operated efficiently, and should indicate opportunities for improvement.

Motivating Interests of the Air Force

Senior officials in the Department of Defense and the military services are stressing the importance of developing better ways of assessing military capability and efficiency [41]. There are four basic questions which motivate this inquiry: (1) what level of military capability can the services achieve with resources available; (2) what capability is required and where are the shortfalls; (3) what resource acquisitions or redistributions are needed to gain maximum improvement in efficiency and effectiveness; and (4) how can management systems be changed to improve the identification and correction of factors which limit the readiness of our military?

Several initiatives have been undertaken during the past few years to improve the way the Department of Defense measures readiness and capability [25, 29, 39, 41].

One basic problem has been the inability of the military to quantify and to establish proven relationships among the various logistics factors which affect performance. There also exist important problems in defining the meanings and interrelationships among terms such as readiness, capability, effectiveness and efficiency.

For purposes of this discussion, capability can be defined as the maximum combat activity that one can reasonably expect to be produced by military units operating in a particular combat scenario given the available technology, the current levels of resources and the managerial abilities of commanders and supervisors. Effectiveness, on the other hand, relates to the degree of achievement of established capability or readiness goals for either peacetime preparedness operations or wartime combat operations.

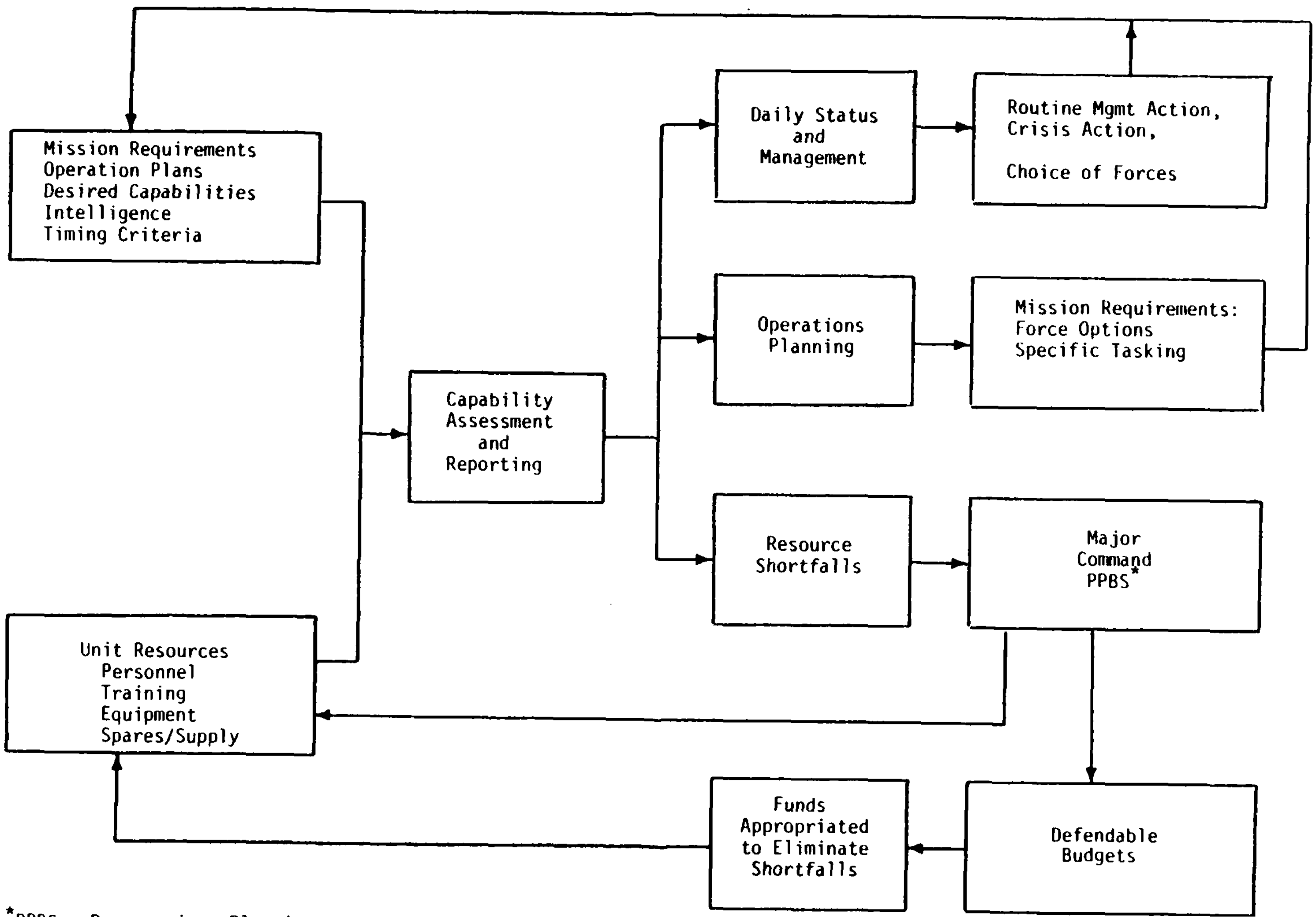
In 1976, General David C. Jones, the Chairman of the Joint Chiefs of Staff, provided the following definition of readiness [33]:

> Total force readiness is difficult to define: From my viewpoint, our state of readiness certainly determines how rapidly and with what effect peacetime forces can be brought to bear upon various crises or conflict situations. It also includes how long and to what degree our forces can be employed. It embodies the capability to successfully accomplish

> tasks within a specified time with current resources and management systems. . . . If there are shortfalls due to limited resources or poor management practices, they should be identified and corrected. This includes a close examination of our plans and readiness reporting systems.

From the key points in General Jones' definition, one might distill the following objectives for measuring and reporting an Air Force unit's combat readiness and capability. Based on an evaluation of a unit's current resources (personnel, training, material, and weapon systems) and management systems, the Air Force must be able to accurately predict and report a combat unit's capability to deploy combat ready forces in the time frames specified by operation plans and to efficiently perform a variety of wartime tasks (combat sorties, airlift, etc.); and, if unit capability is inadequate, to identify limitations or shortfalls so corrections can be made through appropriate management actions or budget programs.

Figure 1.1 provides the author's view of a capability assessment and reporting system in a planning, programming and budgeting context. The system is clearly complex and involves much more than simply the inability of the Air Force to quantify and to establish proven relationships among the various factors which affect



*PPBS = *Programming, Planning and Budget System*

Figure 1.1

Capability Assessment and Reporting for Planning, Programming, and Budgeting

capability. What factors are involved, what data is required, who needs the data and why it's needed, are all important questions.

Of special importance is the section in the lower right hand corner of Figure 1.1 which relates to the problem all military commanders face in justifying to Congress the need for additional dollars for the sake of readiness. For example, in defending their 1976 budget submission, the U.S. Navy proposed that 26.2 million dollars of additional flying hours were needed to improve readiness. Congress refused the Navy's request and indicated that it was unable to discern the readiness deficiency from existing reporting systems [24]. The Air Force has the same problem since its Unit Capability Measurement System (UCMS) falls short of accurately isolating specific resource deficiencies which degrade capability.

Efficiency assessment relates to all of the above definitions of capability, effectiveness and readiness by measuring the degree of achievement of established readiness and capability goals while simultaneously taking into account the degree of resource conservation. These assessments provide information which will aid in the

identification of problems and the elimination of inefficient processes in order to make the best use of available resources.

The budget process is one area which would benefit greatly from efficiency assessments. Budgets which are based on averages of past expenditures in all units, inefficient as well as efficient, and which do not correct for inefficiencies, lead to serious overestimations of resource requirements. In such cases, public money is funding inefficiencies when other worthwhile, efficient programs (military or nonmilitary) could make better use of the funds. Instead, budget requirements should be estimated based on data collected from units operating on the frontiers of efficiency as shown in Figure 1.2, which implies that military services should make every effort to detect inefficiencies and locate frontiers of efficiency.

This does not mean that budgets of inefficient units should be cut arbitrarily. In view of the widespread belief that current military capabilities fall short of that needed to counter the threat, a belief which is represented by the shortfall area in Figure 1.2, it would be unwise to cut budgets until capability reaches acceptable levels. A better approach would be to detect

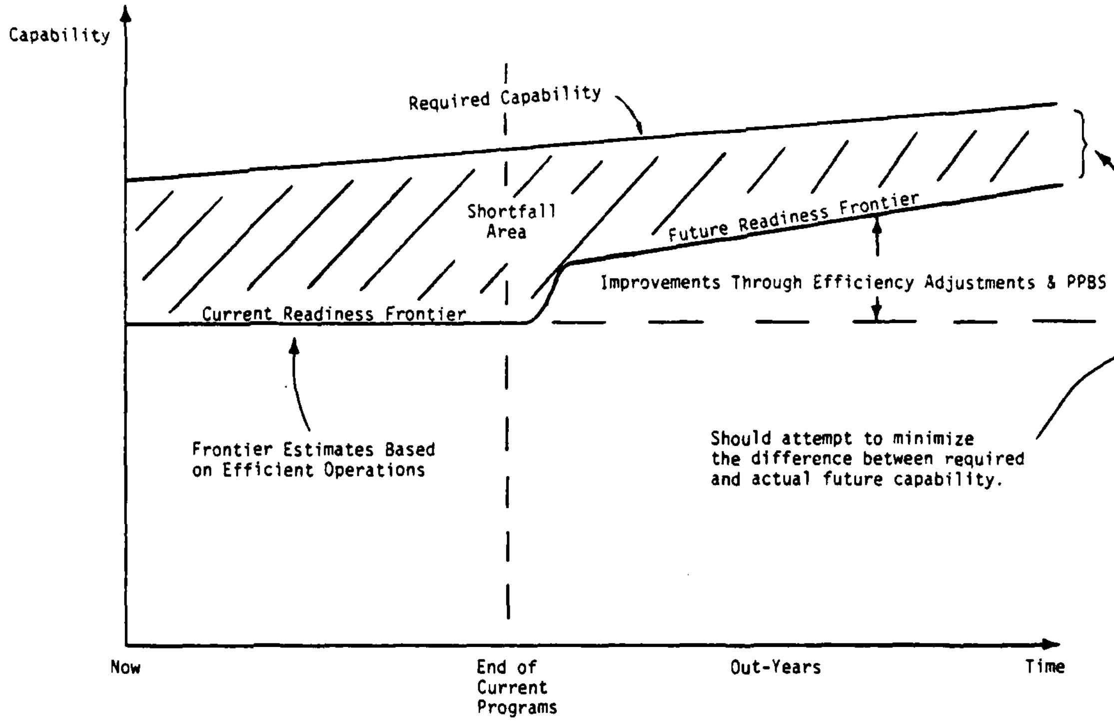


Figure 1.2

Budgeting Based on Frontiers of Efficiency

inefficiencies, adjust substandard units to achieve desired levels of efficiency, and redistribute resources among efficient units to increase overall capability.

Efforts are already underway at the Pentagon to develop a capability based Planning, Programming and Budgeting System (PPBS). Many study agencies are contributing through several parallel initiatives to solve different pieces of the puzzle [41].

One such initiative is the Mission Area Analysis (MAA) program. To quote Lt. Col. Nolte, one of the key analysts during the early stages of MAA development [40],

> Essentially, the Mission Area Analysis program will identify Air Force mission requirements and the capabilities to support them in both current and future years. In those cases where there is not a 100 percent capability, the deficiencies will be identified and prioritized in a logical manner. This prioritized list of needs will influence the way in which resources are funded.

MAA seeks to answer, "What resources will be <u>required</u> to conduct missions which will effectively counter the future threat, and are existing programs adequate to accomplish current and future missions?" On the other hand, Commanders are also interested in the following questions. "Can Air Force units accomplish the missions

presently required by war plans with existing resources? If not, what corrective actions can be taken?"

In other words, MAA should attempt to determine the best, most cost-effective mix of forces (resources) required in future programs to counter projected threats; but capability assessment must also provide managerial information to assess what can be accomplished with the existing mix of forces (resources) regardless of whether the mix is best or not (see Figure 1.2). Of course, both problems must somehow address the relationship between inputs and outputs of decision making units, e.g., squadrons and wings. At the higher levels of command, these efficiency and capability determinations would ideally form the basis upon which allocations of funds and distribution (redistribution) of material assets would be made to produce a more effective and efficient force structure.

Capability information to support corporate planning efforts like MAA is extracted from operational organizations such as shops, squadrons, wings, joint task forces, etc., and aggregated for use in the corporate planning models. These combat capable organizations (decision making units) are charged with the responsibility

of insuring weapon systems and personnel are combat ready. All managers and technicians in these combat organizations must be capability oriented, and they must continually strive to derive from each weapon system its primary mission potential. Supporting analytical models are needed which enable unit level managers and headquarters analysts to assess efficiency and effectiveness of their operations.

It is beyond the scope of this study to provide solutions to all the problems associated with monitoring, reporting and controlling unit effectiveness and efficiency. The foregoing discussion simply provides a backdrop of Air Force problems and interests which serve as motivation for this research.

This study will present DEA theory and extensions which enable the location and analysis of empirically determined frontiers of efficiency for a given set of similar units, e.g., wings. Such analytical capabilities are needed in establishing effectiveness oriented programming, planning and budgeting systems and models in the Air Force.

Problems of Measurement and Evaluation

The Air Force is a public service organization and as such encounters many of the difficulties of

efficiency measurement and evaluation encountered by other public agencies. These difficulties frustrate efforts of problem detection and corrective action.

One of the most important limitations heretofore obstructing the measurement of efficiency in the Air Force has been the fact that absolute measures of efficiency are not practical for combat units. If one were able to specify the maximum achievable sortie production of a tactical fighter wing, given certain levels of resource consumption, then the efficiency of the wing might be measured by dividing the actual sortie production by the maximum achievable sortie production.

But no production function has yet been developed which can forecast maximum sortie potential given the multitude of possible input (resource) combinations and environmental conditions. Thus, the Air Force, in assessing its units, must rely on "relative" measures of efficiency from empirically based comparisons of input and output levels.

To date, the typical approach in assessing unit performance has been to use a large number of "partial" measures (usually ratios and percentages) arranged in tables in a variety of different ways [44]. Managers and

analysts review these tables in hopes of spotting problem areas which can then be more closely scrutinized through follow-on analyses. This review, hampered by the analysts' limited ability to assimilate and simultaneously assess large sets of data, often leads to ill-advised follow-on investigations and erroneous conclusions.

Furthermore, Air Force units, like other public service organizations, have inputs and outputs which are extremely difficult to define and measure. For example, what does the input "workforce" mean and how should it be measured? How should one measure "effective combat sorties?" An additional complication stems from the fact that multiple inputs and outputs of these units are seldom (if ever) expressed in common units of measure, and the causal relationships linking inputs to outputs have usually not been defined mathematically.

As an outcome of these difficulties, the Air Force, in its evaluations of unit efficiency, frequently relies on measures which are production correlates or surrogates for several variables, where relationships are implied but not proven. For example, the average number of aircraft that are mission capable at any particular time might be used as a comparative measure of the ability

of aircraft maintenance units to keep their aircraft fleets in combat ready condition. Obviously, there are other factors besides maintenance capability (e.g., aircraft reliability and usage) which would affect this average.

Further evaluation difficulties are encountered because of differences in the designed operational capability of units. All military units are designed or engineered to produce a given amount of capability in performing a given set of missions with a given (authorized) set of resources [41]. This variety of missions frustrates attempts to compare the relative efficiency of combat units. For example, it would be very difficult, or perhaps fruitless, to compare the efficiency of a small helicopter detachment and a large strategic airlift wing. On the other hand, given that two alternative units of the same type are designed to perform the same mission (same outputs), then an efficiency comparison would be quite useful in determining which one is best.

As a way of getting around this problem of unit variability, the Air Force often resorts to the use of broad measures, which can be applied to all units, but which have questionable meaning. One such measure is "percent fill" of authorized resources, an input measure

which is the ratio of on-hand assets to authorized assets. While thoughtfully applied, this measure leaves much to be desired. It is possible for two units to have the same percent fill measure while one unit lacks mission essential assets and the other lacks nonessential assets. Furthermore, it is possible for a unit to have all authorized assets and still be inefficient or incapable of performing up to standards.

Environmental conditions should also be taken into account when comparing the performance of Air Force combat units. Severe changes in environmental factors such as temperature and moisture can induce failures in aircraft systems, particularly the electronic ones. Furthermore, aircraft do not fly in severe weather, thus, weather can significantly affect maintenance and sortie production. If efficiency differences are to be adequately explained, conditions such as these must be accounted for.

Before the development of the DEA model, which will be presented in the next section, difficulties such as those described above appeared to be insurmountable, thus limiting the development and use of comprehensive efficiency models in the Air Force and other public agencies. But DEA now offers a way to combine multiple inputs

and outputs into a single measure, and meaningful efficiency evaluations can now be obtained with much less difficulty.

Data Envelopment Analysis (DEA)

In this section, the DEA model and its characteristics will be discussed. There are many publications available which thoroughly document the theory and application of DEA. Only a small part of this reference material will be summarized here, together with a few pertinent observations about the DEA model. See Chapter II for a more thorough review of past work.

The DEA Model

Suppose one wishes to compare the efficiency of n decision making units (DMUs), each of which uses varying amounts of m inputs and produces varying amounts of s outputs. Using notation conventions similar to those used by DEA's developers, A. Charnes, W. W. Cooper and E. Rhodes [19], let:

x_{ij} = the amount of input type i used by DMU j during the period of observation, $i = 1,2,\ldots,m$ and $j = 1,2,\ldots,n$.

y_{rj} = the amount of output type r produced by DMU j during the period of observation, r = 1,2,...,s and j = 1,2,...,n.

x_{ik} = the amount of input type i used by the unit k where $k \epsilon j = \{1,2,\ldots,k,\ldots,n\}$ and unit k is the DMU being evaluated. Each DMU in turn will be evaluated.

y_{rk} = the amount of output type r used by DMU_k.

h_k = the efficiency value sought for DMU_k.

v_{ik} = the multipliers for each <u>input</u> type i which will be determined by solution of the model for unit k.

u_{rk} = the multipliers for each <u>output</u> type r which will be determined by solution of the model for unit k.

The following model formulation is used to determine h_k, the efficiency rating of any specified DMU_k, from among the j = 1,2,...,k,...,n units:

$$\text{maximize } h_k = \frac{\sum_{r=1}^{s} u_{rk} y_{rk}}{\sum_{i=1}^{m} v_{ik} x_{ik}} \tag{1.1}$$

$$\text{subject to } \quad \frac{\sum_{r=1}^{s} u_{rk} y_{rj}}{\sum_{i=1}^{m} v_{ik} x_{ij}} \leq 1, \; j = 1,2,\ldots,k,\ldots n$$

$$u_{rk} v_{ik} > 0 \text{ for every } i, r$$

The solution of this nonlinear, ratio problem can be obtained by transforming it into the following equivalent linear programming primal and dual problems and then solving:

Primal

$$\text{maximize } h_k = \sum_{r=1}^{s} u_{rk} y_{rk} \tag{1.2}$$

$$\text{subject to} \quad \sum_{r=1}^{s} u_{rk} y_{rj} - \sum_{i=1}^{m} v_{ik} x_{ij} \leq 0, \quad j = 1,2,\ldots,k,\ldots,n$$

$$\sum_{i=1}^{m} v_{ik} x_{ik} = 1$$

$$-u_{rk} \leq -\epsilon, \quad r = 1,2,\ldots,s$$

$$-v_{ik} \leq -\epsilon, \quad i = 1,2,\ldots,m$$

where $\epsilon > 0$ is a non-Archimedean (infinitesimal) quantity.

Dual

$$\text{minimize } z_k = \theta_k - \epsilon \sum_{r=1}^{s} s_{rk}^{+} - \epsilon \sum_{i=1}^{m} s_{ik}^{-} \tag{1.3}$$

$$\text{subject to} \quad \sum_{j=1}^{n} \lambda_j y_{rj} - s_{rk}^{+} = y_{rk}$$

$$r=1,2,\ldots,s$$

$$-\sum_{j=1}^{n} \lambda_j x_{ij} - s_{ik}^{-} + \theta_k x_{ik} = 0$$

$$i=1,2,\ldots,m$$

$\lambda_j, s_{rk}^+, s_{ik}^- \geq 0$ for all j, r, and i

θ_k unrestricted.

The mathematical theory and proof governing this transformation can be found in articles by Charnes, Cooper, and Rhodes [18, 19] and will not be repeated in this study. But there are a few model characteristics which are worth noting here.

First, the efficiency measure h_k is a scalar ratio measure. Secondly, the constraints of the primal problem insure that the maximum achievable value of h_k is 1. And, the multipliers, u_{rk} and v_{ik}, will be computed in such a way that the unit being evaluated will receive the highest h_k value possible, i.e., no other feasible values of these multipliers will produce a higher efficiency rating for DMU_k. The requirement that multipliers be positive insures that all inputs and outputs have an effect on the final rating. Furthermore, DEA does not require that outputs or inputs have common scales or units of measurement, an important attribute when dealing with difficulties such as nonmonetary objectives and nonpurchased resources. However, all measured input and output values are required to be strictly positive.

A fact not so readily apparent in the formulation is that two DMUs can be rated efficient even when the patterns or mixes of their inputs and outputs are quite different. Differences in managerial strategy and emphasis are treated fairly by the DEA model. Each unit is compared to others in the set which have similar input/output mixes, i.e., those units in its "neighborhood."

DEA can identify units which are efficient or inefficient relative to a frontier of actual achievement; it can provide a limited number of clues on possible causes from analysis of slack variables and multipliers; and it might be of some help in evaluating alternative management actions but only when managerial judgement plays a dominant role in the decision.

Furthermore, the limited amount of managerial information currently provided by DEA is a major improvement over the inadequate, partial (and sometimes inaccurate) measures of performance which are now typically in use in many public service organizations. In addition to its usefulness as a performance monitoring device, this efficiency analysis tool, augmented by the new theory and models discussed later in this study, opens the door for further development and growth in other areas of planning, resource allocation and decision support.

Evolution into Decision Support

The application of DEA to problems of management in various fields is in its infancy. Enough progress has been made to support an optimistic prognosis for its use in identifying efficient and inefficient management units, but extensions beyond that are still being formulated.

Much of the difficulty of applying DEA to management control and planning comes from the nature of decision making. Improvements in organizational efficiency often require strategic decisions affecting changes in output goals, input mixes or the underlying technologies employed by the organization. Strategic decisions are generally novel, complex and open-ended requiring managers to generate and explore solution possibilities with limited knowledge of the situation [38].

Typically, these decisions have no obvious correct answers and often require "what-if" analyses. In such situations, managers are forced to make qualitative judgements; and the degree to which quantitative, analytical techniques are used largely depends on the amount of knowledge available.

These unstructured, nonquantitative processes are often necessary when decision situations have not been

previously encountered in quite the same form and where there exists no known set of ordered responses. In such situations, managers rely almost entirely on judgement, intuition, and experience. Somewhere between the well-structured and unstructured extremes lies many of the strategic efficiency decisions which could be evaluated advantageously using a combination of judgement and mathematical programming. In such "semi-structured" decision processes, clear analytic relationships can reduce uncertainty and enable experienced managers to focus on variable interactions, interpretations and situational value judgements which are not explicitly represented in the analytic models. Solutions of these semistructured problems might require several iterations of modeling and evaluation, each iteration containing components of both structure and judgement, until a limited number of choices have been examined and an acceptable (usually nonoptimal) alternative has been chosen.

All of these points suggest that production frontier analysts, including those using DEA, must be wary of imposing sophisticated but possibly incomplete analytical models onto semistructured problems. The lack of complete knowledge and understanding of production

processes, typically encountered in organizations without scientifically engineered operations, results in the following familiar frontier analysis difficulties:

1. Identification of a comprehensive set of outputs and inputs depends upon more advanced understanding of production technologies than is the case in most social service applications.

2. Input measures are all treated at the same level in present uses, but they may interact in complex ways. Some inputs are hierarchically related, for example, in means-ends chains. Other inputs are at the same level, but threshold effects on one may constrain the effective use of others in producing outputs.

3. Weak causal relationship between inputs and outputs resulting from inadequate understanding of production processes may be further weakened by inefficiencies in the use of resources even among the most efficient units. This places limitations on the confidence with which resource reallocation decisions can be made.

4. Organizations which are engaged in production of the same outputs with the same kind of resources are likely to be subunits of more complex organizations. Elementary schools, for example, are organized into

districts and Air Force squadrons are within wings. Satisfactory techniques for aggregating efficiencies have not been developed.

5. Knowledge of inefficiencies in an organization calls for diagnosis of the intervening processes by means of which inputs are used to produce outputs. An output-input model may indicate pathologies but gives few hints as to the cause or cure.

All of the above difficulties have prevented progress from being made in identification of production functions in the not-for-profit sector. DEA provides hope for limited progress by circumventing some of the difficulties. It makes no assumptions about industry-wide production functions, but uses empirical observations to measure efficiency relative to local frontiers. No claim needs to be made for demonstrated causality between inputs and outputs since unspecified processes are the causal agent and the model allows for an unknown amount of inefficiency to exist.

But further development is necessary to realize the full potential of DEA as a management tool. To mention only one such area, an additional need of efficiency frontier analysts and researchers is a decision support

system for organizational modeling and analyses which could very well lead to greater understanding of the production process and, in some cases, make possible the estimation of frontiers with parametric and stochastic models.

Decision support work has already begun. Relevant theory has been extended by this study (see Chapter III) and software is being developed for a second stage model.

Semistructured planning decisions of particular interest in this study relate to the questions, "What output goals should an inefficient unit adopt, and what resource levels should it expect to use in the next production period in order for the unit to achieve an efficient rating?" In its present state, DEA computes such "values-if-efficient" for organizational units as follows:

$$\begin{aligned} \hat{x}_{ik} &= h_k^* \, x_{ik} - s_{ik}^{-*}, \quad i = 1,2,\ldots,m \\ \hat{y}_{rk} &= y_{rk} + s_{rk}^{+*}, \quad r = 1,2,\ldots,s \end{aligned} \tag{1.4}$$

where h_k^*, s_{ik}^{-*} and s_{rk}^{+*} are the DEA efficiency and slack values at optimality. But these formulas can produce values which are unreasonable from a decisionmaker's viewpoint.

Now suppose input x_{ik} is nondiscretionary, i.e., management cannot control its changes. Furthermore,

suppose this input has positive slack $s_{ik}^{-*} > 0$ associated with it at optimality. The adjustment to the frontier obtained from the Charnes, Cooper and Rhodes DEA model presented in Section D produces a value-if-efficient of $\hat{x}_{ik} = h_k^* \cdot x_{ik} - s_{ik}^{-*}$ which is less than x_{ik}. Having no control over this input, managers in organization k would consider the prospect of changing to the reduced amount $\hat{x}_{ik}$ to be unreasonable.

An alternative value-if-efficient adjustment will be presented in this thesis, one which determines a frontier point for unit k having the same vector of inputs. This approach differs from the method suggested by Banker, Charnes, Cooper and Shinnar [4] which was to remove slack from nondiscretionary inputs before recomputing efficiency.

The presence of slack or surplus values in large amounts creates another problem which must be solved to increase the value of DEA information. Such large values can cause significant overestimations of efficiency. The fact that slack is present means the organizational unit k being evaluated is not "tightly" enveloped by a frontier facet; i.e., at optimality, either the input vector X_k or the output vector Y_k is <u>not</u> a linear combination of observed values from the frontier facet. In other words,

the organization k is tightly enveloped if all slacks are zero implying:

$$\Lambda^{*T} Y_r = y_{rk} \tag{1.5}$$

$$\Lambda^{*T} X_i = h_k^* x_{ik}$$

$$\Lambda^{*T} \geq 0$$

where $\Lambda^{*T} = (\lambda_1^*, \lambda_2^*, \ldots, \lambda_n^*)$, $Y_r^T = (y_{r1}, y_{r2}, \ldots, y_{rn})$ and

$X_i^T = (x_{i1}, x_{i2}, \ldots, x_{in})$.

Other researchers interested in the application of DEA have encountered this problem of efficiency over-estimation [11, 37, 42], a difficulty which is related to the use of the non-Archimedean ϵ (infinitesimal). From a manager's viewpoint, the situation in which a unit is <u>not</u> tightly enveloped can result in ludicrous efficiency estimates. Dr. Jack Davidson, Superintendent of Tyler Independent School District and member of the Educational Productivity Council (EPC) in Texas, asked ". . . Is it really possible that a school with 45 out of 50 minutes wasted in instructional time can be 99% efficient?"[1] The answer is no.

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[1]The DEA methodology is the primary tool used by the EPC. Dr. Davidson made this remark in the presence of 50 other Texas superintendents during the summer 1981 session of the EPC.

The cause of serious errors in estimation like that noted by Dr. Davidson can best be illustrated by examining the results of a DEA evaluation of the single output, two input case shown in Figure 1.3.

This figure depicts five units, all producing a single output amount of one. Three of the units (A, B and C) would be classified as efficient by DEA; and the remaining two, D and E, would be inefficient since they use greater amounts of input to achieve the same output when compared to the frontier segments.

The information generated by this DEA evaluation is shown in Table 1.1. The points $M_1 = (1/\epsilon, 0)$ and $M_2 = (0, 1/\epsilon)$ are treated as fictitious units. They can and do appear in some optimal bases. Any frontier segment containing one of these units (e.g., M_1 is in the basis of unit E) is artificial in the sense that only one of its basis units is an actual unit.

In the case of unit E, the presence of M_1 in its basis caused a serious overestimation of efficiency and an arbitrarily large fictitious rate of substitution $(-1/\epsilon)$ associated with the segment $\overline{CM_1}$. The case of unit D is preferable. Unit D is fully enveloped by the facet connecting A and B. Its efficiency measure $h_D = .67$ and the

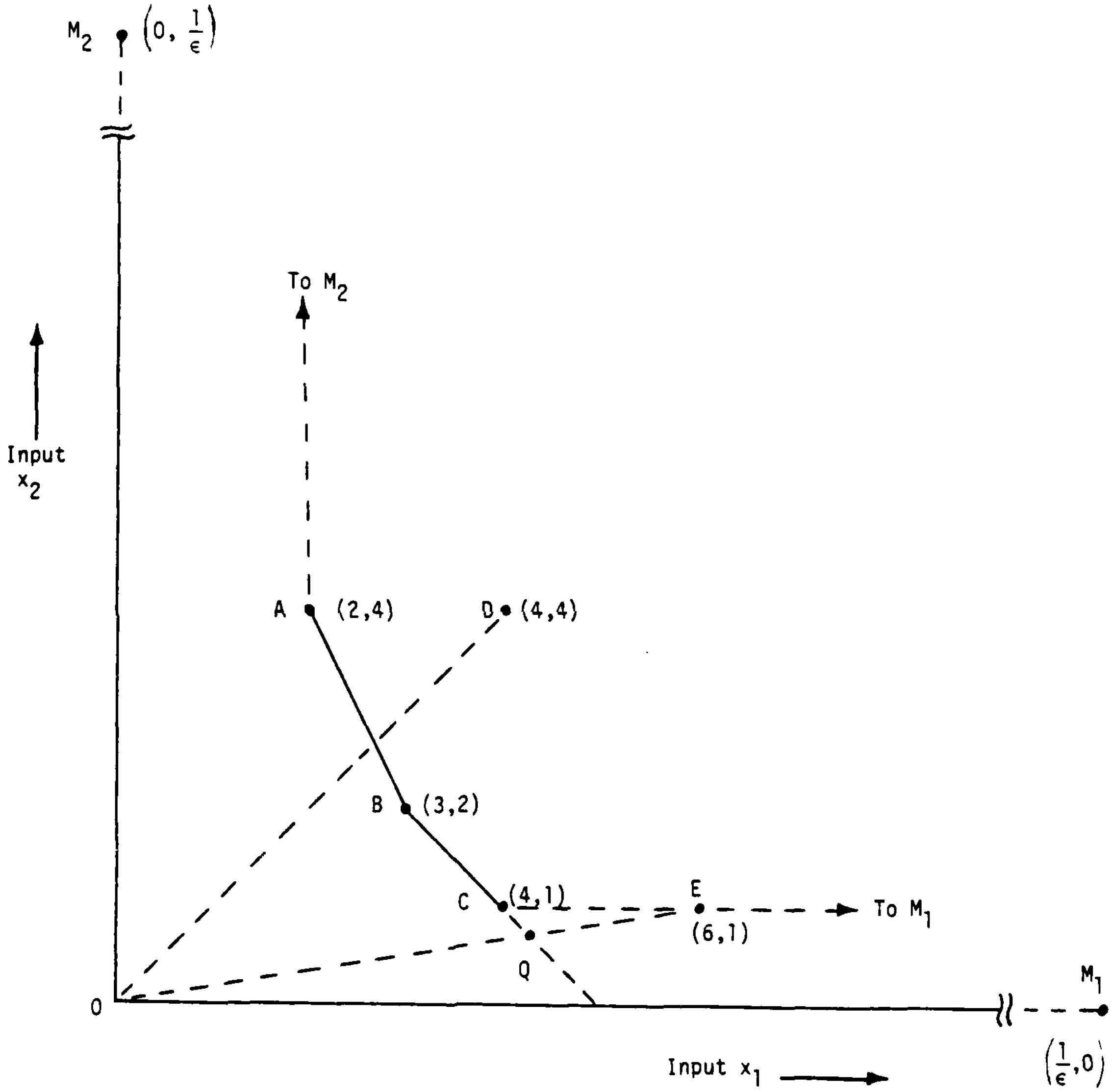

Figure 1.3

Single Output, Two Inputs

TABLE 1.1

DEA Evaluation of the Single Output, Two Input Case

Units	Efficiency (h_k)	Associated Frontier Segments (Basis Units)	Rates of Input Substitution Along Segments $\left(\frac{\partial x_1}{\partial x_2}\right)$	Slack Amounts s^-_{1k}	Slack Amounts s^-_{2k}
A	1.00	$\overline{AB}$ or $\overline{AM_2}$	$-\frac{1}{2}$ or $-\epsilon$	0	0
B	1.00	$\overline{AB}$ or $\overline{BC}$	$-\frac{1}{2}$ or -1	0	0
C	1.00	$\overline{BC}$ or $\overline{CM_1}$	-1 or $-1/\epsilon$	0	0
D	.67	$\overline{AB}$	$-\frac{1}{2}$	0	0
E	1.00	$\overline{CM_1}$	$-1/\epsilon$	2	0

rate of substitution along the segment $\overline{AB}$ are more meaningful and interpretable.

The problem of overestimating unit E efficiency can be dramatically demonstrated by steadily increasing the amount of input x_{1E} of unit E from 6 to 100,004 while holding the output y_E and x_{2E} values constant ($y_E = 1$ and $x_{2E} = 1$) at their original values. This in effect steadily increases the slack variable s^-_{1E} from 2 to 100,000. Figure 1.4 shows graphically that when $\epsilon = 10^{-6}$, an increase in slack from 2 to 100,000 will reduce the efficiency measure h_E from 1.0 to .9. Returning to Dr. Davidson's question, how can a unit which has $y_E = 1$, $x_{1E} = 100,004$ (slack $s^-_{1E} = 100,000$) and $x_{2E} = 1$ be nine-tenths as efficient as one having $y_E = 1$, $x_{1E} = 4$, $x_{2E} = 1$?

A preferred approach would be to measure the efficiency of unit E relative to the extended frontier point Q in Figure 1.3, where Q is a linear combination of nearby efficient units B and C. Of course, units which are fully enveloped like D should be evaluated as before. Furthermore, the fictitious units M_1 and M_2 should be disallowed.

For the simple example described above, this new approach, together with DEA, would provide the information shown in Table 1.2.

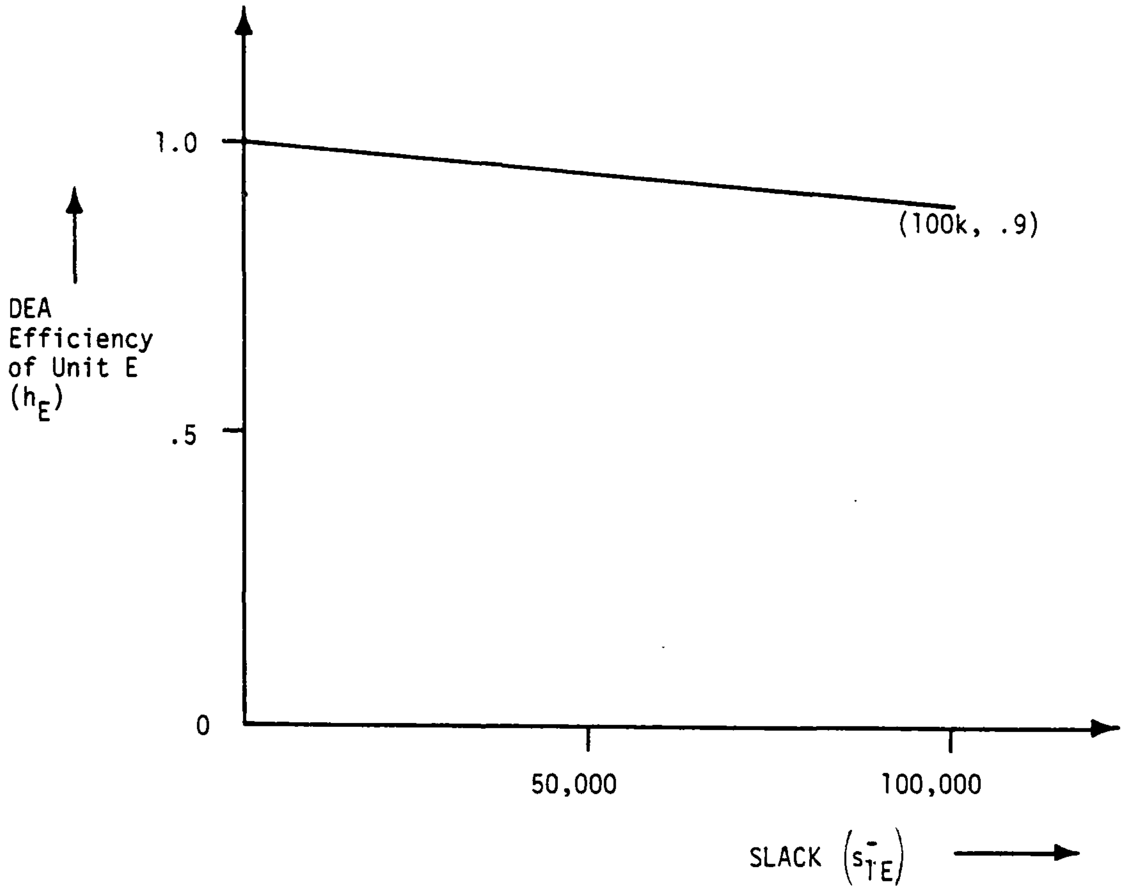

Figure 1.4

DEA Efficiency Decrease Versus Increase of Slack in Input x_1

(at $\epsilon = 10^{-6}$)

TABLE 1.2

New Approach for Efficiency Evaluation

of the Single Output, Two Input Case

Units	Efficiency h_k	Associated Frontier Segments (Basis Units)	Rates of Input Substitution Along Segments $\frac{\partial x_1}{\partial x_2}$	Slack Amounts s^-_{1k}	s^-_{2k}
A	1.00	$\overline{AB}$	$-\frac{1}{2}$	0	0
B	1.00	$\overline{AB}$ or $\overline{BC}$	$-\frac{1}{2}$ or -1	0	0
C	1.00	$\overline{BC}$	-1	0	0
D	.67	$\overline{AB}$	$-\frac{1}{2}$	0	0
E	.71	$\overline{BC}$*	-1*	0	0

*Nearby Facet

In this approach, when units are not fully enveloped, information is drawn from the nearest complete facet in determining the efficiency rating and rates of substitution. The efficiency ratings thus determined are essentially lower bounds in contrast to the "upper bound" overestimation of DEA. For fully enveloped units, these bounds coincide.

Figure 1.5 compares the ratings of unit E obtained from DEA with those of the new approach for increasing amounts of slack. As noted before, DEA is relatively insensitive to increases in slack, but the new approach provides more realistic decreases in efficiency in response to the increases in slack.

Specific Statement of Study Purpose

The example presented in the last section is misleadingly simple. In evaluation of the single output, two input case, one has the advantage of being able to graphically represent the problem and manually compute a solution. Unfortunately, the methods used in that example cannot be automatically generalized to the multiple output, multiple input case.

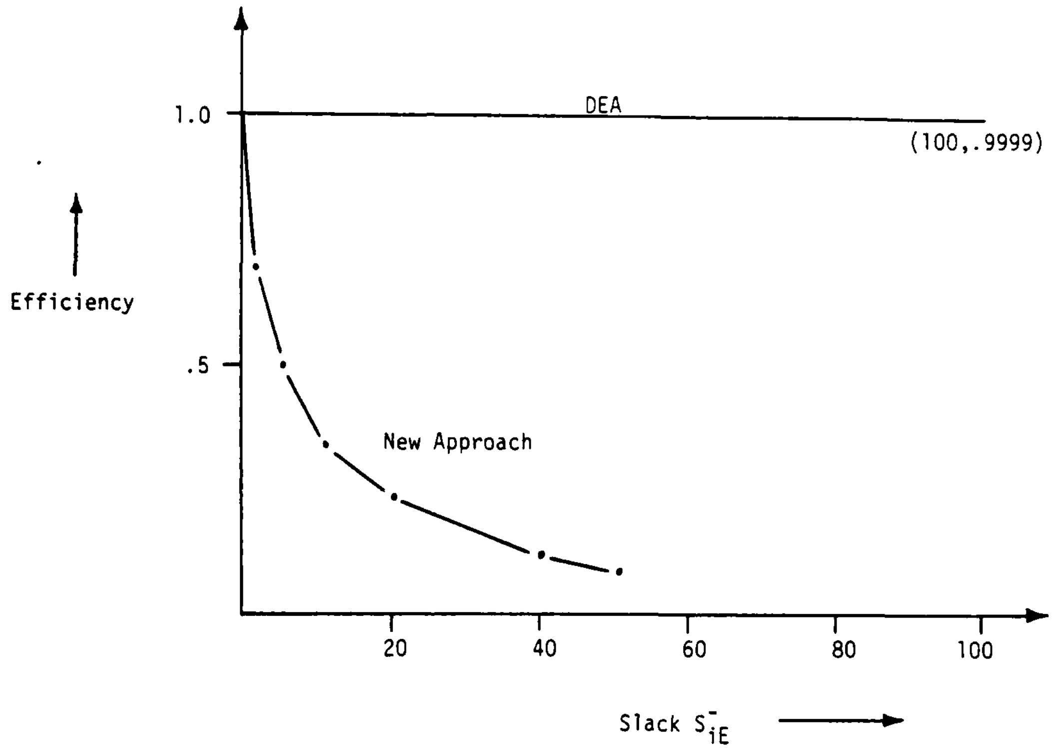

Figure 1.5

<u>DEA (at $\epsilon = 10^{-6}$) Compared to New Approach in Measuring Decreases in Efficiency Caused by Increasing Slack S^-_{iE}</u>

Nonetheless, solution of the aforementioned problems in a multiple input, multiple output situation is absolutely necessary. In all field implementations or demonstrations of DEA known to this author, the presence of significant amounts of slack and the lack of full envelopment of inefficient units is the rule rather than the exception. In fact, the 1982 EPC evaluation of Texas schools revealed that about 60 percent of all schools were inefficient (over 300 inefficient ratings); and all of these units had significant positive slack values and fictitious non-Archimedean vectors in their bases, suggesting of course that the efficiency measures were seriously overestimated.

The fundamental purpose of this study is to eliminate these problems by developing a new second stage model for use in multiple output, multiple input situations, a model which locates a nearby efficient facet as a basis for evaluating each partially enveloped inefficient unit, which calculates the lower bound of efficiency of each such unit relative to this facet, and provides legitimate (nonfictitious) rates of substitution.

The next chapter contains a brief review of the supporting theory and applications relevant to this

purpose. Then, in Chapter III, the new second stage model will be presented together with definitions, properties and proofs including the conditions under which solutions exist as well as alternate ways of generating frontier projections to obtain "values if efficient." In Chapter IV, the new model will be tested on a representative Air Force multiple input, multiple output problem. The efficiency of combat wings will be assessed using DEA and the new second stage model, and the results will be compared to those obtained from use of DEA alone. And, finally, conclusions of the study recommendations for further work will be summarized in Chapter V.

In support of this research, software is under development which will enable interactive data base manipulation and modeling. These new computer capabilities, together with extant DEA and second stage models, will form the basis for the initial prototype of a decision support system. Experience gained from the prototype system will enable managers, analysts, and researchers to make use of efficiency frontier estimation in areas of management planning which heretofore have been unsuitable for application of efficiency models. In any event, the evolution of this decision support system should lead to

more effective management decision making, better control of organizational operations, and increased knowledge of production processes.

CHAPTER II

EXTANT DEA, THEORETICAL BEGINNINGS AND APPLICATIONS[1]

Introduction

There is a small but growing body of literature in the theory and applications of DEA. The model is of particular interest to (but is not limited to) those who study or manage not-for-profit enterprises, since it provides a way to take multiple outputs and multiple inputs into account and to compute an efficiency rating for each unit relative to other units which produce the greatest amount of outputs for their inputs.

Furthermore, DEA does not require that outputs or inputs have common scales or units of measurement, an important attribute when dealing with such difficulties as nonmonetary objectives and nonpurchased resources.

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[1]Much of the material in this section was extracted from an unpublished paper written by this author together with Dr. A. Bessent and Dr. W. Bessent of the University of Texas while all were working under contract with the United States Air Force. This author is greatly indebted to the Bessents for their substantial contributions in organizing and writing this chapter.

Though the literature of DEA has relatively few sources at present, these are somewhat scattered in different journals; and some documents are in technical reports of limited circulation. Thus, a scholar seeking ready access to the methodology does not have an easy entry point. Hopefully this will be remedied by means of the present review.

In the following sections, material will be presented: (1) to relate the theory of Data Envelopment Analysis to its immediate progenitors in the literature of frontier estimation, (2) to contrast DEA with other methodologies currently employed in measuring efficiency, (3) to review the various applications that have been reported and discuss the more intractable problems that have been encountered, and (4) to suggest ways in which DEA could be used for management purposes other than efficiency assessment through extensions of the theory and improvement of existing software.

Theory Development

Introduction

The reader who is familiar with the recent econometric methods ably reviewed by Forsund, Lovell, and

Schmidt [28] will recognize that Data Envelopment Analysis is classifiable as a deterministic, nonparametric model which extends M. J. Farrell's work [27] in that area to solution of the multiinput and multioutput case. DEA differs, however, from the other models reviewed in the purposes for which it has been employed.

Many other models are concerned with industry-wide frontier estimation and only secondarily for measuring inefficiency of individual firms. Stochastic models, for example, frequently provide estimates of efficiency frontiers across firms, and estimates of individual firm inefficiencies are made relative to these across-firm frontiers.

In sharp contrast, DEA is employed chiefly to measure the efficiency of individual firms relative to a frontier neighborhood of technically efficient firms. Thus the production frontier is specific to the firm, rather than to the industry.

Several consequences of this different perspective might be kept in mind when reading the following review. First, DEA results are intended to provide management information for a firm or group of firms rather than to study the production technology of an industry.

Secondly, DEA results vary relative to both the measurements employed and the selection of units comprising the comparison set rather than producing generalizable production functions. Thirdly, DEA permits the incorporation of exogenous factors such as weather that in other models might be treated as random disturbances or perhaps might not be considered at all.

Farrell Efficiency

Farrell's purpose was to provide a satisfactory measure of productive efficiency which takes into account multiple inputs and outputs in a way that would be of use to a wide range of economic statisticians, theorists, policy makers, business persons and civil servants. We will summarize this method for the case where two factors of production (inputs) are used to produce a single product (output). Farrell's explanation of this simple case is briefly paraphrased below because of its relevance to the development and interpretation of DEA.

Figure 2.1 provides a graphic representation of Farrell's efficiency concepts. Curve FF' is an isoquant, which represents the various combinations of inputs, x_1

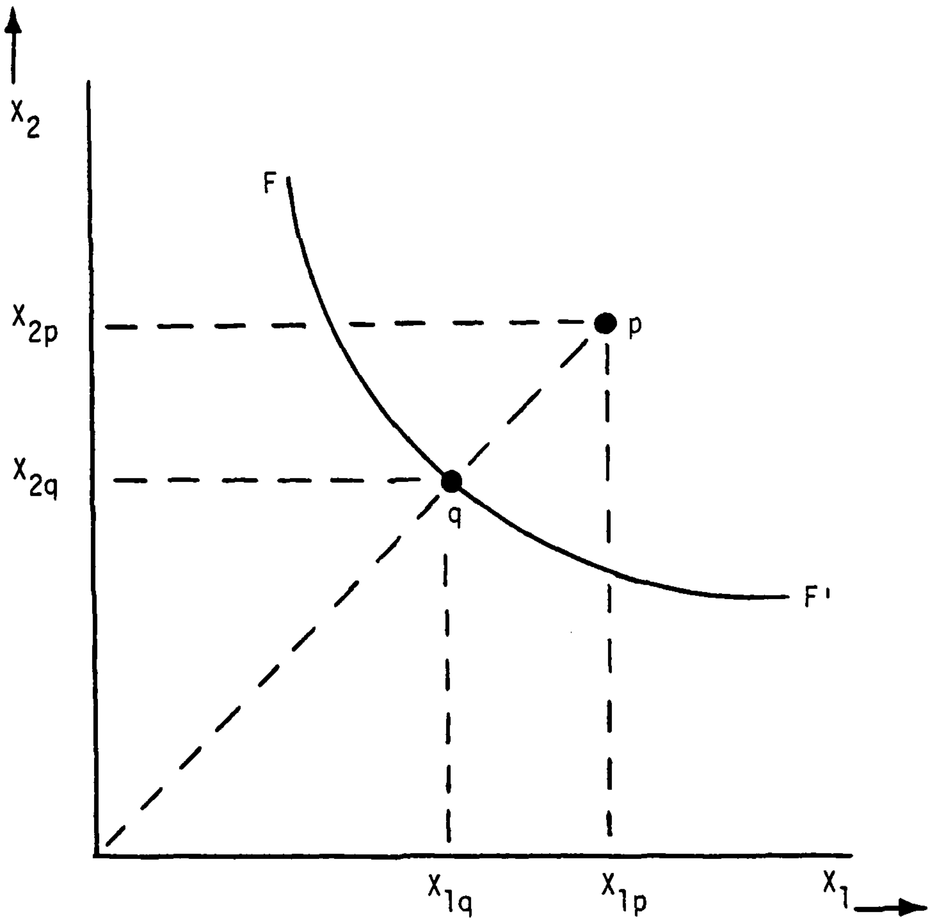

Figure 2.1

Farrell's Efficiency Isoquant

and x_2, which can be used by <u>perfectly efficient</u> organizations to produce <u>one</u> unit of output $y = f(x_1, x_2)$, assuming constant returns to scale.[2] Because perfectly efficient organizations will produce the largest possible output from any given input combination, points between the curve FF' and the origin are unattainable, i.e., FF' is a frontier of production so that it is impossible to obtain a unit of output with combinations in the region between FF' and the origin.

The point p represents the observed inputs per unit output for a particular organization. The point q represents an efficient organization which uses the same proportionate mix of inputs; i.e., if p has the input combination (x_{1p}, x_{2p}), then there exists a real number t such that $(x_{1p}, x_{2p}) = t\,(x_{1q}, x_{2q})$. If we let $\overline{op}$ and $\overline{oq}$ represent the distances from the origin to points p and q respectively, then $t = \overline{oq}/\overline{op}$ which means that an organization represented by q can produce one unit of output with

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[2]An assumption of "constant returns to scale" is required to permit all relevant information to be represented by this diagram; that is, we must assume $f(tx_1, tx_2) = tf(x_1, x_2)$ which implies $1 = f(x_1/y, x_2/y)$. Farrell also discusses economies and diseconomies of scale which could cause measurement difficulties if there are large variations in the level of output produced.

only a fraction, $\overline{oq}/\overline{op}$, of the inputs that p uses. From another viewpoint, one could say that in order for p to be as efficient as q, organization p should produce $\overline{op}/\overline{oq}$ times its current output. Farrell defined the ratio $\overline{oq}/\overline{op}$ as the "technical efficiency" of the organization p.[3]

In order to draw the true frontier isoquant FF', one must know the production function. Unfortunately, frontier production functions are very difficult to derive for complex processes. In Farrell's words, "it is far better to compare performances with the best actually achieved than with some unattainable ideal." That is, one should use an observed standard rather than a theoretical standard when little is known about the true frontier.

Figure 2.2 shows an example of an observed standard in the form of a piece-wise linear frontier SS' and a scatter of points associated with observations from a number of organizations. The relative efficiency of p can now be measured by comparing it to the hypothetical organization q which is a linear combination of frontier points a and b. The technical efficiency again is

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[3]Farrell also defined "price" efficiency and "overall" efficiency measures which are not germane to this discussion but might be of interest to the reader [27].

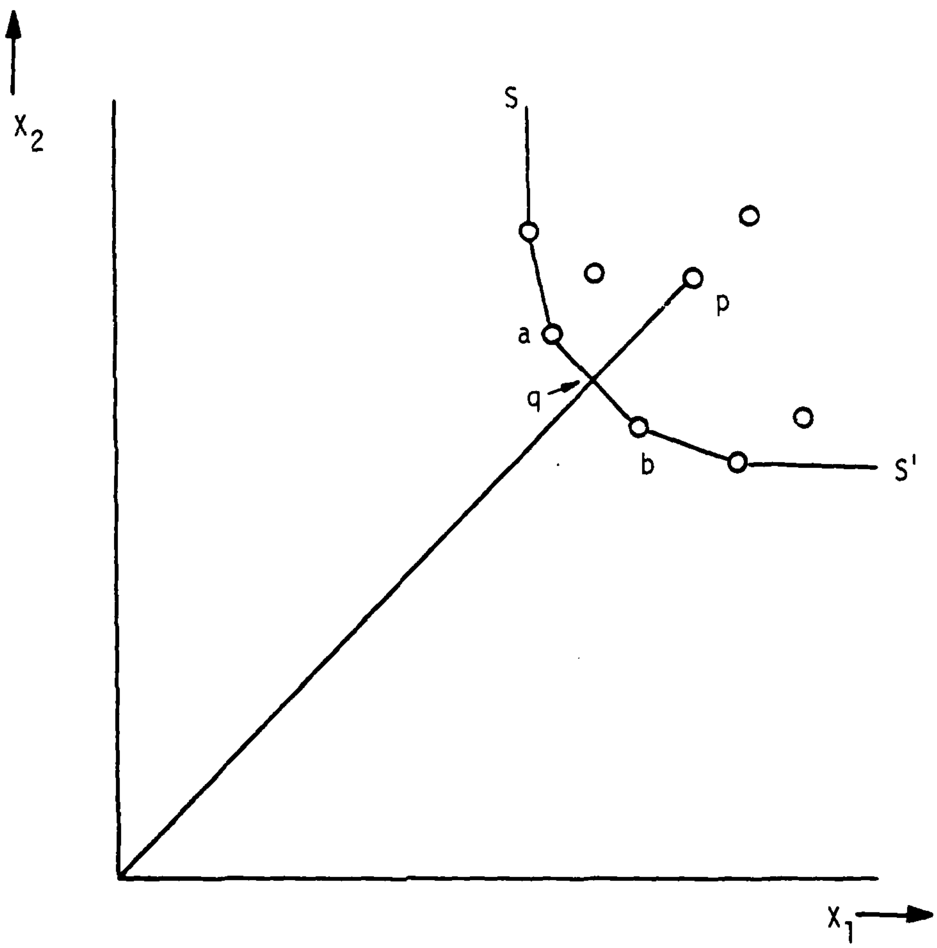

Figure 2.2

Piecewise Linear Efficiency Frontier

represented by $\overline{oq}/\overline{op}$. Farrell first used this technique at a time when linear programming formulations and codes were not available so that the time and cost of computations were prohibitive even for relatively small problems [27].

Regardless of these computational problems, which would later be solved by Charnes, Cooper, and Rhodes [19], the work of Farrell provided a major breakthrough for the single output case specifying a frontier of relative efficiency without designating (assuming) the form of the industry production function.

Subsequent Development of Deterministic Models

If it is reasonable to assume the form of the production function, and if one wishes to make no presumptions about returns to scales, then a second approach might be useful, i.e., compute a parametric convex hull of observed input-output values using a flexible functional form [28]. Aigner and Chu [1], the first to follow Farrell's suggestion, specified a deterministic parametric model for estimating the industry production function for the single output case where the production frontier was

assumed to have a simple but restrictive Cobb-Douglas functional form. The model estimates the parameters of the Cobb-Douglas function with the restriction that observed output values are less than or equal to the estimated values; meaning that all error terms (residuals) are one-sided below the frontier. The functional is fitted to the observations by minimizing either the sum of residuals or the sum of the squared residuals. Although this type of formulation is simple and can accommodate nonconstant returns to scale, it fails to deal adequately with the multiple output characteristics of nonprofit as well as other entities.

On the other hand, Charnes and Cooper with Rhodes ([18], [19], and [43]) first chose the alternate direction of providing new interpretations and extensions of the deterministic, nonparametric Farrell efficiency measure, and then with Banker and Shinnar [4] extended their theory to include deterministic parametric production functions given multiple outputs. Their models overcome the computational difficulties encountered by Farrell, and provide easy treatment of the multiple output case for both types of production function estimation. This was accomplished by redefining efficiency in terms of a nonlinear

programming model which has a linear programming equivalent thereby unleashing the theoretical power and computational capabilities of linear programming for analyzing efficiency and interpreting results. Charnes, Cooper, and Rhodes named this method, Data Envelopment Analysis (DEA).

Banker, Charnes, Cooper, and Schinnar [4] further extended the DEA theory to include parametric efficiency frontier estimation by taking into account multiple output functions which are piece-wise ϕ-linear (includes both log-linear and Cobb-Douglas) thus allowing for increasing, decreasing and constant returns to scale. They also showed that this nonlinear problem is reducible to a finite sequence of linear programming problems and illustrated the method with a numerial example. Since DEA has not yet been applied to industries where the technological possibilities of input to output transformations by operating units is known, this methodology has not been exploited.

None of the models discussed thus far addresses another important theoretical question which needs to be answered in order to improve the potential for using DEA as a basis for decision support and management planning: what <u>feasible</u> input combinations and what <u>realistic</u> output

goals should be adopted by an inefficient organization to insure attainment of the efficiency frontier without decreasing the organization's effectiveness? One would expect that an inefficient organization could reach the frontier by simply reducing inputs or increasing outputs (or both) while maintaining the same relative proportions of inputs and outputs. Unfortunately, in an actual operating environment of an organization, there may be constraints which fix or bound inputs and otherwise limit the discretion of managers.

A "constrained" extension of the DEA model is needed which will enable managers to specify such limitations so that results can be obtained which are both realistic and feasible. Additional decision support requirements also need to be investigated with the intent of providing extended interactive computer capabilities for managerial use. Planners and analysts should be able to interact with efficiency models to test alternative mixes or alternative levels of inputs and outputs.

Comparisons with Related Methodologies

Measurement of efficiency in multiple input, multiple output nonprofit enterprises has been an

econometric problem which has not yielded satisfactorily to other analytical models that have been applied (see [8], [9], [10], [31], and [45]). Reported applications of Data Envelopment Analysis give promise that evaluating efficiency frontiers in certain special cases can be achieved although not with the generalizability sought by those interested in production functions for an industry. Forsund, Lovell and Schmidt's [28] review of frontier estimation methods gives a good account of the strengths and limitations of recently developed econometric models. In contrast to the methods they review, with the exception of Farrell's early work, DEA has not been employed to deal with the problem of industry-wide frontier estimation for which the other econometrics models were developed. For this reason, comparison of DEA with statistical, parametric or stochastic models will not be undertaken here. Instead, this discussion will be confined to the methods that have been compared to DEA results in applications reported thus far: least squares regression and ratio analysis, and to the major concerns common to those techniques. The reader is probably aware of other concerns and unanswered questions pertaining to this discussion. Hopefully, the limited treatment of the subject presented here

will suffice for those who are already well informed about these and other methodologies.

Regression

Ordinary least squares regressions of the single-output, multiple-input variety having both positive and negative error terms produce curves of average relationship [9], [10], [45]. These curves do not represent frontiers, which by definition are based on extremal relations. Actual output values lie above and below the regression curve, and the outputs of efficient units are not necessarily greater than their corresponding regression estimates. With stochastic models, additional information is gained by decomposing residuals, but for frontier estimation in the type of problems which have been addressed by DEA, average estimates are uninformative. Furthermore, in some cases the size and direction of the residuals may appear to have little or no bearing on the efficiency measure (distance from the frontier).

A major difficulty arises in the multiple output case when least squares regression analysis is performed on each output separately [10] and [45]. The other outputs excluded from the analysis have an implicit impact

since they may rely on (compete for) the same resources. Each regression equation might be able to predict adequately an expected level of a single output for an organization, assuming this organization could experience any of the random fluctuations or inefficiencies of the industry (all firms) and recognizing that the influence of other outputs are implicitly taken into account by the deviations from the regression line (residuals). But these equations cannot predict the expected output of an organization whose variations and/or inefficiencies are significantly affected by the given technology and policies of the firm which are not random. Magnitudes of actual outputs of an organization are influenced by both local and corporate policy which may prevent the true expected output values of the organization from conforming to the corresponding regression estimates. Furthermore, there might be little or no correlation between the relative magnitude of actual organizational outputs and the relative magnitudes of their regression estimates, yet relative magnitudes have an important effect on the establishment of frontiers and neighborhoods of comparison in multiple output situations.

If a linear least squares regression equation, with all of its assumptions, is accepted as a proper

representation of organizational productivity, then according to Sherman [45] the "relationships estimated by regression techniques reflect (approximately) efficient input-output relationships." The rate of technical substitution of any two inputs--the ratio of the regression coefficients for those inputs--is assumed constant for all organizations. Moreover, the rate at which an input is transformed into an output is assumed to be the same for every organization.

Under these assumptions, the average output of an organization is not expected to increase unless one or more of the inputs increase; and, if any input is reduced, then another input must increase or the expected output will be reduced. These relationships are those that one would expect to find in organizations having efficient productive capability. In inefficient organizations, either the same output can be achieved with reduced inputs or greater output can be obtained with the same resources.

Perhaps Sherman used the term "approximately" in allowing for the random output variability represented by the regression residual. This variability is assumed to be caused by reasonable, efficient adjustments of output levels in response to random shocks in production or

random market fluctuations. However, if organizations are operating under different technologies, the variances in outputs caused by differences in technical efficiency would be subsumed by this residual term. These variances are not random. Two organizations having precisely the same inputs but different levels of technical efficiency would also have two different <u>expected</u> output levels, and the difference would be accounted for in the residual term. Least squares would consider efficient and inefficient organizations simultaneously and the best fit would be influenced by both types of behavior, including the case where residuals are forced to lie below the frontier [45]. Under such conditions, a single regression equation would misrepresent the productive capability and efficiency of the units.

One would expect that removal of the subsumed difference in technical efficiency from the residual term would produce a regression equation which explains more of the variation in output (higher R^2). Sherman [45] tested this hypothesis with a simulation; and Bessent, Bessent and Clark [9] were able to support his findings by using DEA to identify the efficient organizations in a sample (n=216), then applying least squares regressions to the

efficient units only, and comparing these regression results with the ones obtained when all units were considered.

The R^2 value increased when only efficient units were used in the regression. There were also modest gains in the significance levels of regression coefficients. These improvements were achieved despite the reduction in residual degrees of freedom.

If there exist an adequate number of efficient units in the data set, the above results suggest that one should perform regression (linear or nonlinear) on only the efficient units to obtain the best regression equation, one which comes closer to representing a frontier and which explains a larger portion of output variability.

But, it appears at this point that current regression analysis techniques are largely inappropriate when establishing frontiers for nonprofit organizations which do not have highly mechanistic, scientifically engineered production technologies (see [31]). DEA, on the other hand, provides a useful representation of an attainable production frontier, provides pertinent information about organizational efficiency, and is not subject to the errors and misrepresentations which can result if the

regression assumptions are violated or if the form of the production function is misspecified.

Furthermore, DEA takes all outputs and inputs into account simultaneously including differences in input/output mixes and tradeoffs among factors. It indicates which organizations are on the efficiency frontier, establishes a piece-wise linear approximation of the frontier surface using efficient units, and assigns an efficiency measure based on how far the unit is from a frontier point directly between the unit and the origin, a point for which input and output values are linear combinations of the observations from an efficient set of "neighborhood" organizations. Evaluations of frontier points, neighborhoods, and efficiency measures for individual units are all readily accessible through DEA but are hidden from explicit examination in the regression analysis.

Although DEA appears to be the best alternative for analysis in the public sector, further research is needed to determine how the models of DEA and statistical econometrics can be used in conjunction with one another to improve frontier estimation and analysis. It is likely that a more thorough frontier analysis will be achieved by the use of a number of different but related models.

Ratio Analysis

Ratio Analysis is not a method of frontier estimation, but it is relevant to this discussion because of its frequent use as an ex post facto evaluation tool in analyzing multiple input, multiple output relations. Users of this method examine multiple measures in the form of ratios in an attempt to compare the performance of similar organizations; each ratio typically being a single output measure divided by a single input measure [36] and [45]. Like DEA, ratio analysis is used when the production process is unknown or difficult to model.

Unlike DEA, ratio analyses do not make use of mathematical models to organize or assimilate ratios into a single aggregate measure of efficiency; i.e., they do not simultaneously take into account interactions over the full range of inputs and outputs [45]. As a result, the performances of organizations are difficult to compare using this method particularly when organizations rank comparatively high on some measures and low on others [36].

This difficulty can be illustrated by the following simple example. Consider the two organizations in Table 2.1.

Table 2.1

Difficulties in Using Ratios to Compare Performance

	Organizational Units	
	A	B
Output	1	1
Input 1	1	2.5
Input 2	4	2.5
Ratio 1 (=Output/Input 1)	1	0.4
Ratio 2 (=Output/Input 2)	.25	0.4

Note, Ratio 1 of organization A is larger than Ratio 1 of B, and the situation is reversed for Ratio 2. The relative performance of organizations A and B cannot be determined by examination of these ratios unless the relative importance (weight) of each ratio is specified. Furthermore, as the number of inputs and outputs increases, the problems of weighting and assimilation grow multiplicatively.

Lewin, Morey, and Cook [36] examined this problem in an evaluation of judicial districts. They ranked each of ten output-to-input ratios (2 outputs x 5 inputs) and displayed the number of times that districts were ranked in the upper and lower quartiles over the ten ratio measures. Several districts were noted to have ratios in

both quartiles. Under these circumstances, it would be very difficult to find a simple rule to distinguish efficient districts from inefficient ones without making subjective judgments about the relative importance of each ratio.

Another related difficulty stems from the fact that single ratios provide only partial, incomplete measures of multiple input-output relations, a condition which often leads to incorrect judgments of performance. In actual practice, partial measures such as "units produced per manhour" are used as measures of performance without regard to other inputs such as supplies, fuel, equipment, etc. The data in Table 1 can be used to illustrate the risk in this practice. If one were to compare units A and B based on Ratio 1 alone, unit B would appear to be a better performer than A (.4 > .25), but the reverse would be true if Ratio 2 were considered alone.

Sherman [45] and Bessent, Bessent and Clark [8] used DEA as a vehicle for examining the risks of partial measurements. Each experiment used a set of hypothetical organizations whose inefficiencies were known and detectable by DEA. DEA efficiency evaluations were performed on the sets with all inputs and outputs included. Other

evaluations were performed with one input or output omitted. The results indicated that partial measures can cause misclassifications of efficiency; i.e., organizations might be incorrectly labeled efficient or inefficient, or the magnitudes and causes of the inefficiencies might be misspecified. In general, the omission of relevant inputs or outputs during frontier evaluations may cause distorted neighborhoods of comparison, erroneous slack conditions or measurements relative to the wrong frontier facet.

Despite the above shortcomings, ratios do have the advantage of being familiar to managers and simple to understand [45]. But this advantage is outweighed by the risk of obtaining misleading results unless ratio analysis is used in conjunction with methods of frontier estimation like DEA which are able to take all inputs and outputs into account simultaneously.

Summary of DEA Theory and Review of DEA Applications

Introduction

In the present section, published sources as well as available unpublished papers will be reviewed with the

intent of documenting the state-of-the-art in DEA theory and application. Precursors of DEA were discussed in an earlier section so that attention will be directed here to those studies in which the Data Envelopment Analysis technique of Charnes, Cooper, and Rhodes has been employed.

The review includes both the development of the theory and fields of its reported application. Unsettled questions and issues will be cited where appropriate. First, sources which specified the basic theory are reviewed, then first level applications are cited--those that employ DEA primarily to identify efficient and inefficient management units. Finally, applications which extend DEA are reported. These extensions are limited at present but they concern management uses beyond simply locating inefficient units and point toward reallocating resources, setting output goals, planning to achieve objectives through more efficient operations, and finally, incorporation of DEA into management decision support systems.

The major sources reviewed are displayed in Table 2.1 along with their primary purposes and field of application.

Table 2.2
DEA Literature Citations

	FIELD OF APPLICATION				
Purpose	Theory	Education	Health	Military	Courts
Theory development and/or validation	Charnes, Cooper and Rhodes [19] provided the first publication of DEA programming models. Charnes, Cooper, Lewin, Morey and Rousseau [17] examined problem of non-discretionary inputs. Banker, Charnes, Cooper and Schinnar [4] proposed a variant of DEA for more than one production function.	Charnes, Cooper and Rhodes [18], [19] employed DEA for distinguishing between program efficiency and managerial efficiency in school research.	Sherman [95] validated DEA by expert opinion; and compared it to other methods by simulation.		
Identification of efficient/inefficient units and sources of their inefficiency	Charnes, Cooper and Rhodes [18] distinguished managerial and program efficiencies in public school experiment	Bessent and Bessent [6] studied feasibility of school district application. Bessent, Bessent, Kennington and Reagan [10] studied elementary school units in an urban school district.	Sherman [45] studied efficiency of hospital surgical units.	Charnes, Cooper Devine, Klopp, and Stutz [ ] studied efficiency of army recruiting units over quarterly time series.	Lewin, Morey and Cook [36] studied efficiency of criminal Courts in 30 judicial districts including both cross-sectional and time series data.
Management review of subunits for purpose of resource allocation and/or reallocation		Bessent, Bessent, Charnes, Cooper and Thorogood [7] studied College Occupational-Technical program efficiency for program change and reallocation decisions.		Clark [22] proposed use of DEA for use in capability assessment of Air Force Squadrons and conceptualized decision support system.	

DEA Theory and Mathematical Programming Models

The conceptualization of DEA and formulation of the associated mathematical programming models first appeared in 1978 when Charnes and Cooper and Rhodes' "Measuring the Efficiency of Decision Making Units" [19] was published in the European Journal of Operational Research. Relating their work to isoquant analysis and Farrell efficiency, Charnes, Cooper and Rhodes defined the ratio models for multiple outputs and inputs and reduced the model to its linear programming forms.

In a parallel effort, Charnes and Cooper, and Rhodes employed DEA to make a secondary analysis of data from a large scale social experiment in public education known as Program Follow Through. This work was referred to in several publications by Charnes and Cooper [14], [15], [16], but received its most complete documentation in Rhodes [43] and Charnes, Cooper and Rhodes [18].

The evaluation of the Follow Through experiment was noteworthy in that separate efficiency frontiers were determined for Follow Through and Non-Follow Through schools. This made possible the adjustment of inefficient units to their respective frontiers with the result that

program efficiencies could be compared after removal of what was termed managerial inefficiency of individual units.

Called "inter-envelope" analysis by Charnes, Cooper, and Rhodes, this method of evaluation has not been reported by subsequent researchers although it would seem to have considerable promise for evaluation. One of the unresolved difficulties of this application was the absence of an appropriate statistic for comparing the efficiency envelopes of the contrasted programs. One approach to the problem was suggested by Charnes and Cooper who considered the use of a "divergence statistic" proposed by Kulback [16].

Another problem discovered in early work with DEA was the possibility of having an efficient unit with nonzero slack values. This difficulty was overcome by means of an efficiency theorem reported by Charnes, Cooper, Lewin, Morey and Rousseau [17]. This modification introduced an infinitesimal value (called a non-Archimedean element) to the objective function as shown earlier in this study.

In the same paper, Charnes, et al. [17] considered another intractable problem in the managerial

interpretation of DEA results--that of nondiscretionary resources. That is, some inputs to a DEA application may include exogenous factors beyond the manager's control which nonetheless affect production of outputs. The proposed solution to the problem was to adjust the observations for inefficient units by augmenting outputs according to the first DEA model solution and by similarly decreasing only those discretionary inputs which have controllable slack. A subsequent DEA solution with these adjusted values gives a new efficiency rating after allowing for nondiscretionary inputs. Note that when all inputs are discretionary, then all units are efficient after the adjustment.

An extension of DEA was recently proposed by Banker, Charnes, Cooper and Schinnar [4] in which piecewise ϕ-linear (includes Cobb-Douglas and log-linear forms) production functions are assumed; the problem then becomes one of estimating parameters of the production functions by means of DEA "envelopment conditions" where production function forms are imposed on outputs and ordinary linear programming equivalents are given. The development takes into account possible economies or diseconomies of scale for the multiple output case. Although a numerical example was given, no applications have been reported.

Validation of Data Envelopment Analysis

Lewin and Morey [35] showed that DEA measured efficiency consistent with regression results and known validity, but the only extensive validation of DEA which has been reported was Sherman's study of efficiency of 22 hospital medical-surgical units [45]. Three different evaluations were made: (1) a comparison of DEA with ratio analysis and statistical regression, (2) the use of a panel of health care experts, and (3) review by staff of a hospital which received an inefficient rating.

In Sherman's first validity evaluation, a simulation was employed using hypothetical organizations having known inefficiencies and technologies. It was found that DEA detected the known inefficiencies and provided more managerial information than the partial measures given by ratio analysis. Unlike DEA, regression models produced best fit results which confounded observed inefficiencies and efficiencies.

Results of DEA were compared to judgments of four hospital administrators who gave qualified agreement on 7 out of 10 hospital inefficiency ratings and strong disagreement about one case. The strong disagreement was

later resolved by in-depth study. However, the experts continued to believe that some hospitals rated efficient were really inefficient and that, in general, they preferred simpler modes of analysis to DEA.

Finally, a field study of one inefficient hospital was conducted in which hospital management verified the data and related the actual operations of the medical-surgical unit to DEA results. Inefficiencies detected by DEA included slack inputs for supplies, beds and staff all of which were verified by the hospital administrators and it was concluded that the inefficiency rating was valid and that the sources of inefficiency were identifiable.

DEA Application in Various Fields

Most of the reported applications of DEA have been for the purpose of demonstrating that efficient and inefficient management units can be identified by means of the technique and that inputs and outputs can be defined which have meaning for management audit and review. These can be called "applications of the first level" to distinguish them from management planning and reallocation uses.

In the present section, first level applications in health care, education, military, and court systems are reported. The singular use of DEA by Charnes, Cooper and Rhodes [18] for program evaluation was reviewed in the previous section.

Health Care Application. Sherman's study [45] is the only application reported in the health care field. Teaching medical-surgical units in 22 Massachusetts hospitals were chosen for the purpose of comparing DEA measurement of technical efficiency with other methods. Available data from an annual report submitted to the State Rate Setting Commission were used to select three inputs and four outputs for the analysis. The outputs represented both patient care and training functions of the hospitals, thus providing greater complexity than would be the case if only nonteaching hospitals were included. Surgical units were selected, rather than the hospital as a whole or other subunits such as radiology, in order to decrease the variability in case mix.

As reported earlier, the hospital application demonstrated that DEA is potentially useful for measurement of the relative technical efficiency of hospital subunits. As might be expected, however, the most serious

limitation concerned the specification and measurement of inputs and outputs since the ones in actual use in hospitals are not typically in forms which are sufficiently accurate for DEA use. In addition, the application was limited to only one type of subunit in the complex organization of a comprehensive hospital. More extensive usefulness remains to be demonstrated.

Application in Public Education. Two similar applications of DEA in large urban school districts were reported by Bessent and Bessent [6] and Bessent, Bessent, Kennington and Reagan [10].

In the first of these, 55 elementary schools comprised the units of analysis with standardized achievement test scores providing two measures of output. Thirteen inputs were chosen as proxy measures of attendance, socioeconomic status and mobility, resources allocated to the schools and indicators of organizational climate within schools.

The second application in a different city studied both elementary and secondary schools but results for only 167 elementary schools were reported. The purpose of the second study was to investigate the use of DEA to provide a better comparative analysis of academic performance

than was previously available to the administration and Board of Education.

Data employed by the district for their school comparisons provided input and output variables for DEA: reading and mathematics achievement test scores for third and sixth grade (4 outputs), resources allocated (seven inputs) and five nonallocatable inputs relating to students' characteristics and prior years' achievement. In addition to the usual reports of efficiency and slack, results were displayed by plotting achievement against efficiency. This provided information about both effectiveness and efficiency of schools.

Results of the Houston study were seen as useful not only for management audit but also for system scanning information to use for balancing scarce resources among schools. The latter use was not pursued in the study however, since DEA models for reallocation decisions did not exist at that time.

As in the Sherman study [45], limitations encountered were inadequate specification of outputs and inputs, and difficulties in communicating DEA methods and results to users. However, some progress in reducing these difficulties through the formation of a network of school

districts was reported. Personnel of member schools provided better input and output measures and attended training sessions on the use of DEA results for improved planning. Since the study, three years of successful operation of the network have been achieved.

DEA Applications in the Military. No military applications of DEA have been reported in the periodical literature although some work for the U.S. Army by Charnes, Cooper, Devine, Klopp, and Stutz [ ] is in progress and Navy recruiting districts have been studied by Lewin and Morey [35].

The military recruitment studies were similar in purpose to those already discussed in that they sought to measure the efficiency of recruiting centers and to provide information for superordinate review of sources of inefficiency. They differed, however, in that time series data were employed for the analysis. The study in progress by Charnes, et al. [ ] used quarterly data to provide a series for each unit in the analysis. Lewin and Morey [35] used monthly data to compare the efficiency of a single recruiting district with itself at different time periods. As might be supposed, a significant seasonal effect was observed for recruiting.

The inclusion of a time series of measurements is an important extension since any serious management employment of DEA would result in repeated measures over time. Procedures have not been reported, however, for using time series information for more than repeated snapshots.

Potential use of DEA for management of Air Force maintenance squadrons was discussed by Clark [22]. This application envisioned DEA as a component in a decision support system. It will be discussed later along with other extensions of DEA.

The Measurement of Efficient Operation of Courts. Lewin, Morey and Cook [36] employed DEA to evaluate the administrative efficiency of 100 superior courts in 30 North Carolina judicial districts. They employed log-log stepwise regression of variables from archival data to select five inputs and two outputs for DEA computations. Taking the perspective of the court administrator, DEA was shown to be superior to ratio analysis in that it provided a single efficiency rating and required no subjective appraisal of multiple performance ratios. In addition, DEA was viewed as useful as a diagnostic tool when used in combination with field audits.

Management Decision Making

Identifying inefficient units is only a prelude to the desire to remove sources of inefficiency. For the manager of a unit who seeks to make more effective use of resources, DEA provides descriptive information for needed output augmentation and/or resource conservation. Two reported applications of DEA for planning and managerial decision making will now be considered.

The first was a study reported by Bessent, Bessent, Charnes, Cooper, and Thorogood [7] which resulted in actual management decisions. It concerned the evaluation of proposed program modifications in the occupational-technical division of a college.

The problem addressed was to determine the effect on efficiency of existing programs contingent upon various changes proposed by subordinate program heads: augmenting new technologies in an existing program, introducing a new competing program, or abolishing an inefficient program and redistributing its resources.

Three outputs and four inputs were selected from data used for budgeting and planning for 22 different programs under the direction of the division administrator. DEA solutions were first obtained for the existing

programs and inefficient units were identified. Their planning alternatives were identified on the basis of consumer demand, employment trends and allocatable space.

One proposal evaluated was to augment the Business Technology program by adding legal secretarial training. The Director evaluated a plan for rescheduling classes and faculty to better utilize resources. With these changes and with meeting projected contact hour goals, a new DEA evaluation resulted in an improved efficiency rating. On the basis of these findings, a decision was made to expand the program.

A similar change was evaluated in which an already efficient program was augmented. A subsequent DEA analysis showed that such augmentation would change the efficiency frontier causing a reduction in the comparative efficiency rating of some other units. The Director decided to implement this change in order to provide a higher motivation for existing subprogram heads with the result that overall productivity would increase.

A third change was based on DEA evaluation of proposals for three new cost centers. Following this analysis, the Director decided against one because it was less efficient than existing programs. The other two had favorable recommendations.

Finally, two existing programs which had efficiency ratings of less than .58 were candidates for discontinuation. Both were older programs for which the job market had declined. Three alternatives were evaluated by using DEA to determine the impact of phasing out the inefficient programs and redistributing allocatable resources to other programs.

The use of DEA for evaluating proposed program changes was found to be feasible but was limited by the absence of an overall planning model.

The feasibility of designing a decision support system based upon DEA was examined by Clark [22] in an analysis of management systems for maintaining Air Force readiness. A semistructured decision support system was seen as a way to improve the quality of decisions through a more systematic decision process and increased managerial understanding and control of operations.

Clark proposed the rudimentary decision steps shown in Figure 2.3. It begins with DEA efficiency ratings, progresses to use of other information obtained from the linear programming solutions, and finally evaluates decision alternatives.

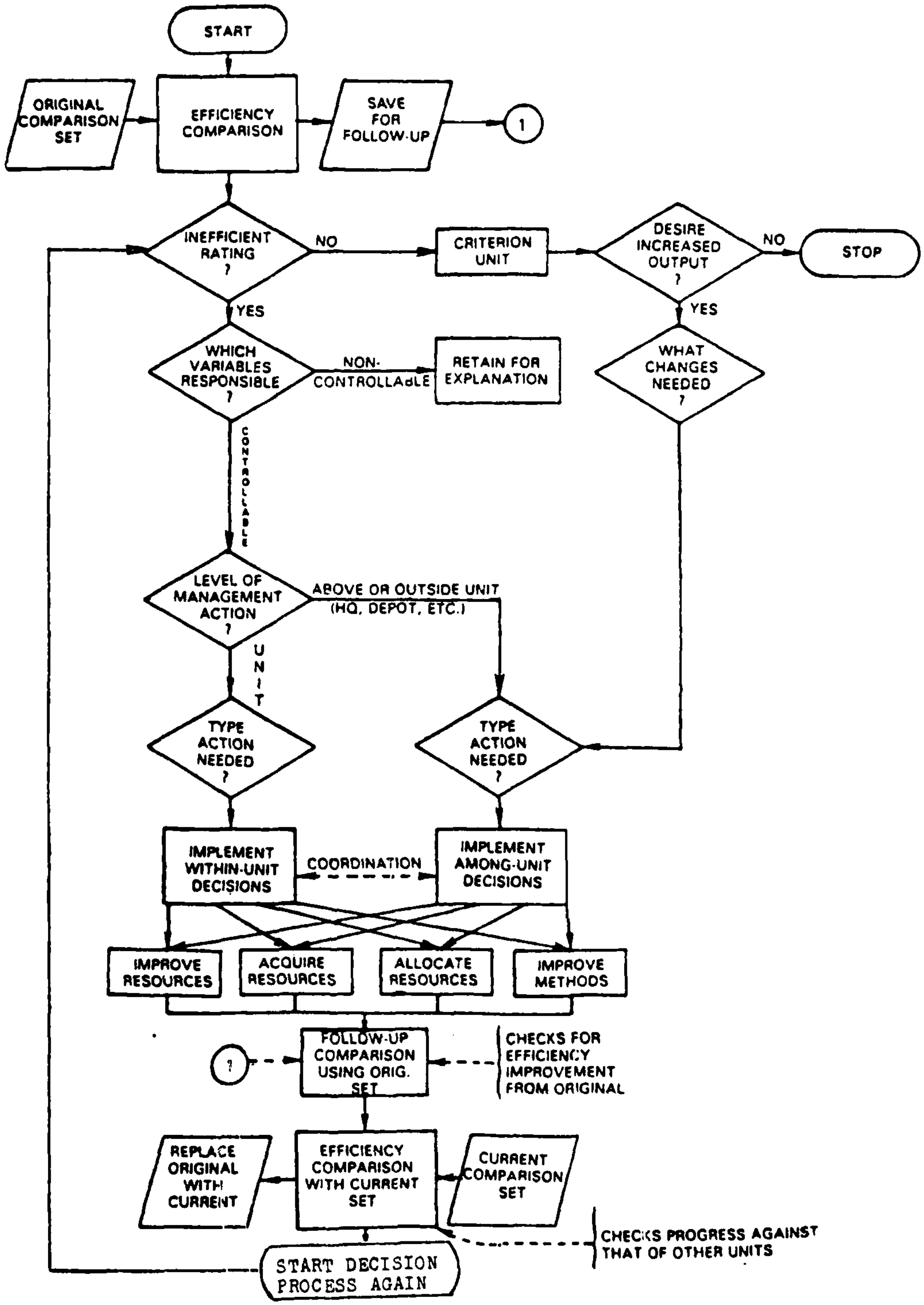

Figure 2.3

Efficiency Decision Steps

According to Clark, managers may monitor and control the efficiency of their organizational units by means of these steps which correspond to the following set of management questions.

a. Is the unit efficient when compared to others having similar missions and technologies?

b. If the unit is rated inefficient, which variables are responsible for the low rating? Are there excesses in inputs, shortfalls in outputs, or combinations of both?

c. If rated efficient, and if an increase in outputs is desirable, which inputs can be increased to achieve greater output?

d. To what extent are the desired changes in variables controllable by management? For example, weather and other external causes which might reduce efficiency are not controllable.

e. If variables are completely or partially controllable, which responsibility centers must be involved in taking the appropriate corrective actions?

f. What actions are appropriate? Some problems are caused by operational or resource deficiencies

which are strictly within the management control of the unit itself. Others are caused by inadequate support from other agencies upon which the unit must rely. Is the indicated action to: (1) improve the use of existing resources (manpower, equipment, supplies, facilities, information), (2) acquire additional resources, (3) allocate/reallocate resources (budgets), or (4) improve production methods (plans, schedules, reorganizations, policies, procedures, etc.)?

g. Have the management actions resulted in the desired efficiency changes? A follow-up efficiency evaluation would be useful after corrective actions have had suf ient time to take effect. This evaluation should compare the unit's efficiency to the original comparison set (paragraph 1 above).

h. Is the unit efficient when compared to current data from other units in the comparison set? This would measure progress made versus that achieved by others, and it begins the cycle again.

Implementation of the DEA-based decision support system would require additional mathematical models, would

be enhanced by an interactive modeling language and would depend upon the development of more precise logic for relating managerial inquiries to the results of an efficiency analysis. The extensions presented in the next chapter are crucial parts of this development. DEA-based management planning for removal of inefficiencies and for allocations to improve effectiveness are not possible unless one is able to specify target frontier regions or facets of efficiency for each unit and determine the rates of substitution within these facets.

AD-A139 430 DATA ENVELOPMENT ANALYSIS AND EXTENSIONS FOR DECISION SUPPORT AND MANAGEMENT PLANNING(U) AIR FORCE INST OF TECH WRIGHT-PATTERSON AFB OH C T CLARK MAY 83 2/3

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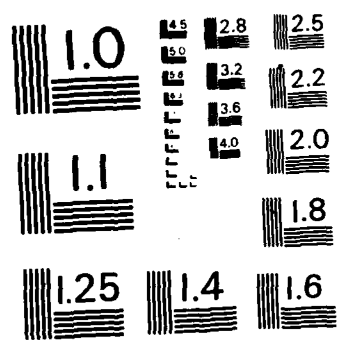
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CHAPTER III

THEORETICAL EXTENSIONS

Introduction

In this chapter, theoretical extensions are presented to overcome the aforementioned difficulties in DEA resulting from use of non-Archimedean infinitesimals (ϵ values). The solution of the Charnes, Cooper and Rhodes DEA problem will provide information for use in performing second stage evaluations to locate the nearest empirical facet of each inefficient unit and, if necessary, to extend frontier neighborhoods in an attempt to achieve full envelopment (no slack) of all such units. Units identified by DEA as having slack in inputs or outputs will receive a new efficiency measure which essentially provides a lower bound of efficiency. Each of these new efficiency values will be measured relative to an extended frontier point which is a linear combination of the actual input and output observations of efficient units, and, if possible no slack variables will be permitted in the optimal basis.

Furthermore, extended frontier points will provide alternative estimates of input and output "values if

efficient" for comparison with those provided by DEA. In computing these values for an inefficient unit, one can either hold the unit's output vector of observations constant and multiply its input values by the efficiency measure, or hold its input vector constant and multiply outputs by the reciprocal of the efficiency measure. Even when such input and output projections are not deemed technically feasible (or achievable) by managers, the second stage evaluation, which reveals a complete basis of efficient units in a nearby facet, can be used in conjunction with DEA in developing more realistic planning estimates.

Each of the above characteristics are presented below in greater detail. But before proceeding with the necessary definitions, properties and proofs related to the second stage model, the primal and dual formulations of the DEA model will be recalled. A familiar notational form has been used (see [6] [19]), and the models have been numbered so that they can be referred to easily in subsequent paragraphs. Recall that:

x_{ij} = the amount of input type i used by DMU j during the period of observation, $i = 1,2,\ldots,m$ and $j = 1,2,\ldots,n$

y_{rj} = the amount of output type r produced by DMU j during the period of observation, r = 1,2,...,s and j = 1,2,...,n

x_{ik} = the amount of input type i used by the unit k where $k \in J = \{1,2,\ldots,n\}$ and unit k is the DMU being evaluated. Each DMU in turn will be evaluated.

y_{rk} = the amount of output type r used by DMU_k

h_k = the efficiency value sought for DMU_k

v_{ik} = the multipliers for each <u>input</u> type i which will be determined by solution of the model for unit k

u_{rk} = the multipliers for each <u>output</u> type r which will be determined by solution of the model for unit k

As indicated in Chapter I, the following model formulations are used to determine h_k, the efficiency rating of any specified DMU_k, from among the j = 1,2,..., k,...n units:

<u>Primal</u>:

$$\text{Max } h_k = \sum_{r=1}^{s} u_{rk} y_{rk} \tag{3.1}$$

$$\text{s.t.} \quad \sum_{r=1}^{s} u_{rk} y_{rj} - \sum_{i=1}^{m} v_{ik} x_{ij} \leq 0 \quad j = 1, 2, \ldots, n$$

$$\sum_{i=1}^{m} v_{ik} x_{ik} = 1$$

$$-u_{rk} \leq -\epsilon \quad r = 1, 2, \ldots, s$$

$$-v_{ik} \leq -\epsilon \quad i = 1, 2, \ldots, m$$

where $\epsilon > 0$ is a non-Archimedean infinitesimal quantity.

<u>Dual:</u>

$$\text{Min} \quad \theta_k - \epsilon \sum_{r=1}^{s} s_{rk}^{+} - \epsilon \sum_{i=1}^{m} s_{ik}^{-} \qquad (3.2)$$

$$\text{s.t.} \quad \sum_{j=1}^{n} \lambda_j y_{rj} - s_{rk}^{+} = y_{rk}$$

$$r = 1, 2, \ldots, s$$

$$-\sum_{j=1}^{n} \lambda_j x_{ij} + \theta_k x_{ik} - s_{ik}^{-} = 0$$

$$i = 1, 2, \ldots, m$$

$\lambda_j \geq 0$ for all j; s_{rk}^{+}, $s_{ik}^{-} \geq 0$

θ_k unrestricted

Definitions

Two definitions and a definitional property are now presented to clarify the meaning of the terms "proper facet" and "fully enveloped," terms which are used frequently in subsequent developments.

Definition. The frontier facet defined by empirically observed basis units is called a proper facet if it is formed by j = 1,2,...,s+m-1 efficient units (actual units from original comparison set) and if there exist column vectors $\Phi > 0$ and $\Psi > 0$ such that

$$(\Psi^T, \Phi^T) \begin{pmatrix} Y_j \\ X_j \end{pmatrix} = 0 \qquad (3.3)$$

for every efficient unit j in the optimal basis of some inefficient unit k where

$$Y_j^T = (y_{1j}, y_{2j}, \ldots, y_{sj})$$

and

$$X_j^T = (x_{1j}, x_{2j}, \ldots, x_{mj})$$

The hyperplanes

$$I(X_j) = \{X^T = (x_1, x_2, \ldots, x_m) \mid \Phi^T X = \Phi^T X_j > 0\} \quad (3.4)$$

for basis units $j = 1, 2, \ldots, s+m-1$ are called the input hyperplanes defined by the efficient facet. Similarly,

$$O(Y_j) = \{Y^T = (y_1, y_2, \ldots, y_s) \mid \Psi^T Y = \Psi Y_j\} \quad (3.5)$$

$$\text{for } j = 1, 2, \ldots, s+m$$

is the efficient facet's family of output hyperplanes.

If we define the constant $C_j \equiv \Phi^T X_j = \Psi^T Y_j$ for each j in the basis, the input and output hyperplanes of basis unit j can be rewritten:

$$\phi_1 x_1 = C_j - \phi_2 x_2 - \phi_3 x_3 - \ldots - \phi_p x_p - \ldots - \phi_m x_m$$

$$(3.6)$$

$$\psi_1 y_1 = C_j - \psi_2 y_2 - \psi_3 y_3 - \ldots - \psi_q y_q - \ldots - \psi_s y_s$$

Since $\phi_i > 0$, $i = 1, 2, \ldots, m$ and $\psi_r > 0$, $r = 1, 2, \ldots, s$ in a proper facet, we have

$$\frac{\partial x_1}{\partial x_p} = -\frac{\phi_p}{\phi_1} < 0 \quad (3.7)$$

$$\frac{\partial y_1}{\partial y_q} = - \frac{\psi_q}{\psi_1} < 0$$

In general, we can explicitly represent any input or output hyperplane j as follows:

$$\begin{aligned} \phi_i x_i &= C_j - \Big(\sum_{p \neq i} \phi_p x_p \Big) \\ \psi_r y_r &= C_j - \Big(\sum_{q \neq r} \psi_q y_q \Big) \end{aligned} \qquad (3.8)$$

so that

$$\begin{aligned} \frac{\partial x_i}{\partial x_p} &< 0 \quad i \neq p \quad i,p \in \{1,2,\ldots,m\} \\ \frac{\partial y_r}{\partial y_q} &< 0 \quad r \neq q \quad r,q \in \{1,2,\ldots,s\} \end{aligned} \qquad (3.9)$$

In other words, all of the hyperplanes of a proper facet, $I(X_j)$ and $O(Y_j)$, $j = 1,2,\ldots,s+m-1$, have negative slopes; i.e., on the frontier, inputs trade off with other inputs and outputs trade off with outputs.

Furthermore, the relations at (3.3) and (3.8) above imply the following two relations for proper facets:

$$\psi_r y_r = \sum_{i=1}^{m} \phi_i x_i - \sum_{q \neq r} \psi_q y_q$$

$$\frac{\partial y_r}{\partial x_i} = \frac{\phi_i}{\psi_r} > 0 \qquad (3.10)$$

for $i = 1,2,\ldots,m$ and $r = 1,2,\ldots,s$. In other words, in order for an efficient unit to remain on the frontier, incremental increases (decreases) in the inputs must produce corresponding increases (decreases) in the output levels achieved.

In actual practice, empirically derived relative frontiers of efficiency might not exhibit all of the above characteristics of a proper facet. Empirically formed, relative frontiers of efficiency represent what actually happened and not what should have happened. As a result, such relative frontiers may fall short of the theoretical ideal and fail to conform to what one would expect from an absolute frontier with its positive marginal productivities and negative rates of substitution.

It is important to detect when such failures occur and why. Perhaps positively correlated, nonsubstitutable input measures were selected for use in the DEA model, a situation which could lead to positive rates of substitution. Or perhaps an input was chosen at the

outset of the analysis based on the assumption that an increase in its amount <u>should</u> produce a corresponding increase in one or more of the outputs without knowing whether it <u>will</u> or not in all frontier regions. In this case, subsequent frontier evaluation might reveal that in one or more facets this input exhibits negative correlation with the outputs, thus failing to conform to the requirement of having positive marginal productivities in proper facets.

This situation actually occurred in the 1980-1981 DEA evaluation of Texas Schools performed by the Educational Productivity Council. One input, the percent of teachers having more than three years experience, was shown to be negatively correlated with the output measures of reading, writing and math in at least one frontier neighborhood, an undesirable situation because experienced teachers as a rule are paid more than their less experienced colleagues since educators believe that increased experience should lead to better teaching methods.

The above violations of the characteristics of a proper facet will cause zero or negative multipliers in the second stage evaluation to be discussed later. But

before proceeding with the second stage methodology, the DEA condition of being fully enveloped and its relationship to the existence of a proper facet need to be defined and clarified.

Definition. An inefficient unit k is "in the cone" or "fully enveloped" if DEA yields an optimal solution to (3.2) having all s_{ik}^- and s_{rk}^+ slack variables[1] nonbasic and equal to zero i.e. each basic variable in (3.2) is a λ_j which corresponds to actual vectors of observed inputs and outputs for some frontier unit j.

If unit k is totally enveloped, the requirement that all slack variables be nonbasic means there are s+m-1 actual units which define the inefficient unit k's neighborhood frontier facet. Further, the λ's associated with these units form unit k's optimal basis. Let the s+m-1 elements of $J^* = \{j \mid \lambda_j$ is in the optimal basis of unit k$\}$. Furthermore, let

$$V_k^{*T} = (v_{1k}^*, v_{2k}^*, \ldots, v_{mk}^*) > 0 \text{ and}$$

---

[1]In DEA literature the values of these variables are referred to as slack values because of the way in which Charnes, Cooper and Rhodes first defined the DEA model [19]. In order not to be confusing, these variables will be referred to as slack variables here, but their counterparts in the second stage model will be called surplus variables as is usually the case.

$$U_k^{*T} = (u_{1k}^*, u_{2k}^*, \ldots, u_{sk}^*) > 0$$

represent the solution vectors of multipliers at optimality for the DEA primal problem (3.1). If the vectors $\Phi \equiv V_k^*$ and $\Psi \equiv U_k^*$, then the DEA solution requires that for all s+m-1 elements $j \in J^*$

$$\Psi^T Y_j - \Phi^T X_j = U_k^{*T} Y_j - V_k^{*T} X_j = 0, \qquad (3.11)$$

i.e., the basis J* defines a proper facet. The converse follows immediately from the definition of a proper facet. Thus, the following definitional property summarizes this connection between full envelopment and proper facets.

Property 1. In DEA, an inefficient unit k is fully enveloped if and only if its optimal basis defines a proper frontier facet.

This somewhat trivial property was presented here simply to make the point that in DEA the existence of a proper frontier facet for unit k and the condition of unit k being fully enveloped are synonomous. Such clarification will be of benefit later when the new second stage evaluation is discussed which measures the

efficiency of not fully enveloped units relative to extensions of nearby proper facets.

Second Stage Method for Inefficient, Not-Fully-Enveloped Units

A new second stage method is presented in this section, one which can be used in conjunction with DEA in evaluating the range of inefficiency in not-fully-enveloped units. The DEA evaluation is performed first, and its solution effects the objective function and the form of the constraints in the initial iteration of the second stage evaluation. Of course, if a unit is fully enveloped, its proper facet is reflected in the DEA solution and no second stage evaluation is required.

Post-DEA evaluation of not-fully-enveloped units will enable the determination of a lower bound of efficiency by overcoming the problems discussed in Chapter I (pages 27 - 35) resulting from use of an arbitrarily small value to estimate the infinitesimal ϵ which heretofore caused slack values to have negligible impact on efficiency measures resulting in serious overestimations of efficiency, particularly when significant amounts of slack were present. In fact, the DEA efficiency can be thought of as an upper bound efficiency

measure since by definition the infinitesimal ϵ values are extremely small relative to the magnitudes of input and output observations.

In contrast to DEA, the second stage evaluation measures the efficiency of k relative to nearest <u>proper</u> facet and its extended hyperplanes thereby providing a measure of minimum efficiency which is sensitive to the amount of slack present. It is minimum in the sense that further reduction in the efficiency measure would require a change in basis corresponding to selection of a more distant facet. Each of these characteristics will be clarified in the discussions that follow.

For this discussion let $E_k^{(1)}$ = the set of DEA efficient units associated with the basis of unit k ($\lambda_j^* \geq 0$) and let $\overline{E}_k^{(1)}$ = the set of DEA efficient units not associated with the basis of unit k ($\lambda_j^* = 0$). The superscript (1) identifies the first iteration of the second stage model. The complete collection of DEA frontier units is $E = E_k^{(1)} \cup \overline{E}_k^{(1)}$. This reduced set E is the reference set for all of the second stage iterations, the first of which is shown below.

$$\text{Max } h_k^{(1)} = \sum_{r=1}^{s} \mu_{rk}^{(1)} s_{rk}^{+*} + \sum_{i=1}^{m} \nu_{i..}^{(1)} s_{ik}^{-*} \tag{3.12}$$

$$\sum_{r=1}^{s} \mu_{rk}^{(1)} y_{rj} - \sum_{i=1}^{m} \nu_{ik}^{(1)} x_{ij} = 0 \quad \text{for } j \in E_k^{(1)}$$

$$\sum_{r=1}^{s} \mu_{rk}^{(1)} y_{rj} - \sum_{i=1}^{m} \nu_{ik}^{(1)} x_{ij} \leq 0 \quad \text{for } j \in \bar{E}_k^{(1)}$$

$$\sum_{i=1}^{m} \nu_{ik}^{(1)} x_{ik} = 1$$

$$\mu_{rk}^{(1)}, \nu_{ik}^{(1)} \geq 0 \quad r = 1,2,\ldots,s \text{ and}$$

$$i = 1,2,\ldots,m$$

where s_{rk}^{+*}, s_{ik}^{-*} are slack values at optimality from the DEA dual problem (3.2).

The dual of problem (3.12) is:

$$\text{Min } \omega_k^{(1)} \tag{3.13}$$

s.t.

$$\sum_{j \in E_k^{(1)}} \lambda_j^{(1)} y_{rj} + \sum_{j \in \bar{E}_k^{(1)}} \gamma_j^{(1)} y_{rj}$$

$$- s_{rk}^{(1)} = s_{rk}^{+*}, \; r = 1,2,\ldots,s$$

$$x_{ik}\omega_k^{(1)} - \sum_{j \in E_k^{(1)}} \lambda_j^{(1)} x_{ij} - \sum_{j \in \bar{E}_k^{(1)}} \gamma_j^{(1)} x_{ij}$$

$$- s_{ik}^{(1)} = s_{ik}^{-*}, \; i = 1,2,\ldots,m$$

$$\omega_k^{(1)},\ \lambda_j^{(1)} \text{ unrestricted; } \gamma_j^{(1)},\ s_{rk}^{(1)},\ s_{ik}^{(1)} \geq 0$$

The way in which the above formulations depend on the previous DEA optimal solution guarantees that problems (3.12) and (3.13) have feasible solutions. The solution $\mu_{rk} = u^*_{rk}$ for $r = 1,2,\ldots,s$, $\nu_{ik} = v^*_{ik}$ for $i = 1,2,\ldots,m$ is feasible for problem (3.12) where u^*_{rk} and v^*_{ik} are optimal values from DEA. The solution $\lambda_j^{(1)} = \lambda_j^*$ for $j \varepsilon E_k^{(1)}$, $\gamma_j^{(1)} = \lambda_j^* = 0$ for $j \varepsilon \overline{E}_k^{(1)}$, $s_{rk}^{(1)} = y_{rk}$ for every r, $s_{ik}^{(1)} = 0$ for every i and $\omega_k = \theta_k^*$ is feasible for (3.13) where λ_j^*, $j = 1,2,\ldots,n$ and θ_k^* are optimal values from the DEA dual model given in (3.2). Thus, existence of feasible solutions for both the primal and dual problems guarantees the existence of a finite optimum.

Furthermore, if unit k is inefficient and not fully enveloped, i.e., positive slack values exist in the optimal solution to DEA dual problem (3.2), then the values of the above solution which were derived from DEA primal problem (3.1) and dual problem (3.2) can be substituted in the objective functions of (3.12) and (3.13) to yield the following two equalities:

$$h_k^{(1)} = \sum_{r=1}^{s} \mu_{rk} s_{rk}^{+*} + \sum_{i=1}^{m} \nu_{ik} s_{ik}^{-*} = \sum_{r=1}^{s} u_{rk}^{*} s_{rk}^{+*}$$

$$+ \sum_{i=1}^{m} v_{ik}^{*} s_{ik}^{-*} \tag{3.14}$$

$$\omega_k^{(1)} = \theta_k^{*} \tag{3.15}$$

Because of complementary slackness, if $s_{rk}^{+*} > 0$ or $s_{ik}^{-*} > 0$ for some r and i, then the corresponding primal multipliers equal their lower bound, i.e., $u_{rk}^{*} = v_{ik}^{*} = \epsilon$. Furthermore, if $u_{pk}^{*} > 0$ or $v_{qx}^{*} > 0$ for some p and q, then their associated slack values, s_{pk}^{+*} and s_{qk}^{-*}, equal zero. Thus, the equality at (3.14) can be rewritten as:

$$h_k^{(1)} = \sum_{r=1}^{s} u_{rk}^{*} s_{rk}^{+*} + \sum_{i=1}^{m} v_{ik}^{*} s_{ik}^{-*} = \sum_{r=1}^{s} \epsilon \cdot s_{rk}^{+*}$$

$$+ \sum_{i=1}^{m} \epsilon \cdot s_{ik}^{-*} \tag{3.16}$$

Since the objective function of the DEA dual problem (3.1) and primal problem (3.2) are equal at optimality and since h_k^* and θ_k^* are both greater than zero:

$$\sum_{r=1}^{s} \epsilon \cdot s_{rk}^{+*} + \sum_{i=1}^{m} \epsilon \cdot s_{ik}^{-*} = \theta_k^{*} - h_k^{*} < \theta_k^{*} \tag{3.17}$$

Combining relations (3.15), (3.16) and (3.17), one obtains:

$$h_k^{(1)} = \theta_k^* - h_k^* < \theta_k^* = \omega_k^{(1)} \tag{3.18}$$

Thus, the solution derived from the DEA values for inefficient unit k which is not fully enveloped is not optimal for the first iteration of the second stage model because the objective function values of the primal and dual model are not equal, i.e., $h_k^{(1)} < \omega_k^{(1)}$.

To achieve optimality, one or more of the μ's and ν's associated with positive s_{rk}^{+*} or s_{ik}^{-*} in the objective function must enter the basis at a positive amount. Entrance of any such multiplier would drive its associated surplus variable ($s_{rk}^{(1)}$ or $s_{ik}^{(1)}$) in dual model (3.13) from the basis which in turn would require one of the $\gamma_j^{(1)}$ variables to enter the basis[2] and replace the leaving surplus variable, except when entrance of a $\gamma_j^{(1)}$ is not feasible. The case where $\gamma_j^{(1)}$ variables will not enter is discussed later.

---

[2]All λ_j associated with $E_k^{(1)}$ were basic in DEA and will remain so in every iteration of the second stage model. Thus, $\gamma_j^{(1)}$ variables are the only nonbasic variables other than surplus variables which could enter.

The equality constraints associated with the set $E_k^{(1)}$ in problem (3.12) guarantee that the new optimal solution will contain the λ_j variables associated with the units in the original DEA neighborhood plus others that entered as the surplus variables were replaced. If all surplus variables $s_{rk}^{(1)}$ and $s_{ik}^{(1)}$ in (3.13) are non-basic, the process stops and the proper facet has been determined. If at least one $\gamma_j^{(1)}$ enters the basis and if at least one of the surplus variables remain basic then another iteration is required. For this next iteration, let

$$J^{(1)} = \{j \in \bar{E}_k^{(1)} \mid \gamma_j^{(1)} \text{ enters basis in iteration } (1)\} \tag{3.14}$$

and let

$$E_k^{(2)} = E_k^{(1)} \cup J^{(1)} \tag{3.20}$$

then the second iteration primal model becomes:

$$\text{Max } h_k^{(2)} = \sum_{r=1}^{s} \mu_{rk}^{(2)} s_{rk}^{(1)*} + \sum_{i=1}^{m} \nu_{ik}^{(2)} s_{ik}^{(1)*}$$

$$\text{s.t.} \quad \sum_{r=1}^{s} \mu_{rk}^{(2)} y_{rj} - \sum_{i=1}^{m} \nu_{ik}^{(2)} x_{ij} = 0 \text{ for } j \in E_k^{(2)}$$

$$\sum_{r=1}^{s} \mu_{rk}^{(2)} y_{rj} - \sum_{i=1}^{m} \nu_{ik}^{(2)} x_{ij} \leq 0 \text{ for } j \in \bar{E}_k^{(2)}$$

$$\sum_{i=1}^{m} \nu_{ik}^{(2)} x_{ik} = 1$$

$$\mu_{rk}^{(2)}, \nu_{ik}^{(2)} \geq 0$$

As shown in the first iteration, a feasible solution can be constructed for iteration (2) from the optimal values of iteration (1), a solution which falls short of optimality. This second iteration model will attempt to drive some of the remaining surplus variables $s_{rk}^{(2)}$ or $s_{ik}^{(2)}$ from the basis of the dual model replacing them with $\gamma_j^{(2)}$ variables. The iterative process continues until an iteration (N) is reached where s+m-1 $\lambda_j^{(N)}$ and $\gamma_j^{(N)}$ variables are basic or where no $\gamma_j^{(N)}$ can enter in which case a leaving surplus variable $s_{rk}^{(N)}$ or $s_{ik}^{(N)}$ is replaced by another surplus variable prohibiting further progress in obtaining a full basis associated with s+m-1 actual units. The form of the primal and dual models at iteration (N) are shown below and numbered for future reference.

Primal

$$\text{Max } h_k^{(N)} = \sum_{r=1}^{s} \mu_{rk}^{(N)} s_{rk}^{(N-1)*} + \sum_{i=1}^{m} \nu_{ik}^{(N)} s_{ik}^{(N-1)*} \tag{3.22}$$

$$\text{s.t.} \quad \sum_{r=1}^{s} \mu_{rk}^{(N)} y_{rj} - \sum_{i=1}^{m} \nu_{ik}^{(N)} x_{ij} = 0 \quad \text{for } j \in E_k^{(N)}$$

$$\sum_{r=1}^{s} \mu_{rk}^{(N)} y_{rj} - \sum_{i=1}^{m} \nu_{ik}^{(N)} x_{ij} \leq 0 \quad \text{for } j \in \overline{E}_k^{(N)}$$

$$\sum_{i=1}^{m} \nu_{ik}^{(N)} x_{ik} = 1$$

$$\mu_{rk}^{(N)}, \ \nu_{ik}^{(N)} \geq 0$$

Dual

$$\text{Min } \omega_k^{(N)} \tag{3.23}$$

$$\text{s.t.} \quad \sum_{j \in E^{(N)}} \lambda_j^{(N)} y_{rj} + \sum_{j \in \overline{E}^{(N)}} \gamma_j^{(N)} y_{rj} - s_{rk}^{(N)}$$

$$= s_{rk}^{(N-1)*} \qquad r = 1,2,\ldots,s$$

$$x_{ik}\omega_k^{(N)} - \sum_{j \in E^{(N)}} \lambda_j^{(N)} x_{ij} - \sum_{j \in \overline{E}^{(N)}} \gamma_j^{(N)} x_{ij} - s_{ik}^{(N)}$$

$$= s_{ik}^{(N-1)*} \qquad i = 1,2,\ldots,m$$

$$\omega_k^{(N)}, \ \lambda_j^{(N)} \text{ unrestricted}; \ \gamma_j^{(N)}, \ s_{rk}^{(N)}, \ s_{ik}^{(N)} \geq 0$$

When conditions are not favorable for entry of γ variables into the basis, a proper facet cannot be formed because one of the μ or ν multipliers in the primal objective function would need to assume a negative value to enable another $\gamma_j^{(N)}$ to enter the basis. A negative multiplier would violate the characteristics of a proper facet which require negative rates of substitution and positive marginal productivities. One might wish to continue the iterations allowing negative multipliers to measure the lack of substitutability of variables or the syphoning effect that a poorly used resource has on production.

Other properties of the iterative model will be described in the next section to illustrate the additional frontier information and analytical capabilities that can be obtained from this second stage process.

Other Properties

The primary objective of this section is to clarify what is meant by the concepts "minimum efficiency" or "lower bound of efficiency," concepts which were introduced in previous sections. The meaning of these terms will first be examined geometrically. Then the

mathematics of the second stage evaluation will be explained relative to this geometric interpretation.

Minimum Efficiency Measure Explained Geometrically

Consider the single output, two input case shown in Figure 3.1. In this figure there are four actual observations (A, B, C, and K) and only two proper facets, the line segments $\overline{AB}$ and $\overline{BC}$. The dotted lines represent all possible extended facets, real and artificial, for the not-fully-enveloped inefficient unit K, facets which could be used to provide a measure of efficiency for unit K.

Clearly, the extended facet connecting A and M_2 would provide the smallest efficiency measure $(\overline{OT}) \div (\overline{OK})$. But A is the farthest frontier point from K and M_2 is not an actual observed point on the frontier. The segment $\overline{CM_1}$ provides the greatest efficiency measure $(\overline{OP}) \div (\overline{OK})$, but it also contains an artificial unit. This measure is the one DEA would choose. DEA would also assign an mount of slack equal to $\overline{CP}$, and C would be the only frontier unit represented in the DEA basis for unit K. For these reasons, when the unit being evaluated is

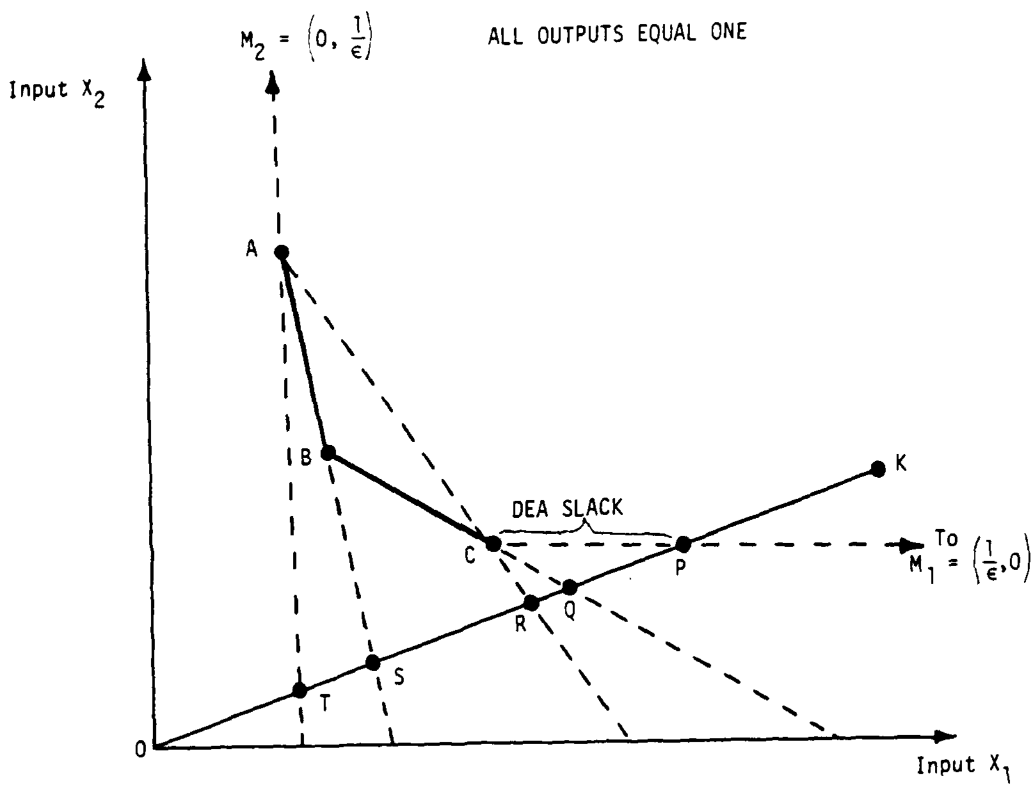


Figure 3.1

Locating the Nearest Proper Facet and Measuring the Minimum Efficiency of Unit K

not fully enveloped, DEA is said to produce an upper bound efficiency measure and an artificial frontier facet.

The line passing through A and C is not a frontier facet and therefore does not provide a desirable reference for measuring the efficiency of unit K. Of the remaining two lines, the extension of proper facet $\overline{BC}$ is preferred to $\overline{AB}$ because $\overline{BC}$ comes closer to enveloping K and it contains the observation C which is the closest of all frontier points to K.

The efficiency measure $(\overline{OQ}) \div (\overline{OK})$ is called the "minimum efficiency." For not-fully-enveloped units, it is always less than the DEA measure, and it is the smallest efficiency measure achievable without changing to a reference facet farther from K. Furthermore, the "true" relative frontier of efficiency lies somewhere between the dotted line extending $\overline{BC}$ and the segment $\overline{CM_1}$ *if* unit C is truly efficient.

The measure $(\overline{OQ}) \div (\overline{OK})$ is most desirable because it makes use of information from actual frontier units; moreover, the rates of substitution and marginal productivities in the neighborhood of extended frontier point Q are also available from the nearby proper facet $\overline{BC}$.

One final observation is necessary to complete this geometric treatment. Note that as K moves closer to full envelopment through reduction of the slack amount $\overline{CP}$, the measures $(\overline{OQ}) \div (\overline{OK})$ and $(\overline{OP}) \div (\overline{OK})$ move closer to equality. Thus, the difference between the upper and lower bounds of efficiency, as defined in this study, measures the degree of nonenvelopment. For fully enveloped units, the upper and lower bounds are equal and the DEA optimal solution provides an appropriate efficiency measure.

Mathematical Expression for the Minimum Efficiency Measure

The mathematical relationships of the second stage method can now be examined in light of the above geometric properties. First, consider the following optimal dual relations for the rth output constraint at each of N iterations (last to first):

Iteration	Dual Constraints at Optimality
N	$\sum_{j \varepsilon E^{(N)}} \lambda_j^{(N)*} y_{rj} + \sum_{j \varepsilon \overline{E}^{(N)}} \gamma_j^{(N)*} y_{rj} - s_{rk}^{(N)*} = s_{rk}^{(N-1)*}$

Iteration	Dual Constraints at Optimality
N-1	$\sum_{j \varepsilon E_k^{(N-1)}} \lambda_j^{(N-1)*} y_{rj} + \sum_{j \varepsilon \bar{E}_k^{(N-1)}} \gamma_j^{(N-1)*} y_{rj}$ (3.24)
	$- s_{rk}^{(N-1)*} = s_{rk}^{(N-2)*}$
$\vdots$	$\vdots$
1	$\sum_{j \varepsilon E_k^{(1)}} \lambda_j^{(1)*} y_{rj} + \sum_{j \varepsilon \bar{E}_k^{(1)}} \gamma_j^{(1)*} y_{rj} - s_{rk}^{(1)*} = s_{rk}^{+*}$
0(DEA)	$\sum_{j=1}^{n} \lambda_j^{*} y_{rj} - s_{rk}^{+*} = y_{rk}$

Also recall that at any iteration M, the vectors associated with $j \varepsilon E_k^{(M-1)}$ in the previous iteration are forced to remain in the basis because of the equality constraints imposed in the primal. Thus,

$$E_k^{(N)} \supseteq E_k^{(N-1)} \text{ and } \bar{E}_k^{(N-1)} \supseteq \bar{E}_k^{(N)} \qquad (3.25)$$

which guarantees that each successive iteration contains the original members of the DEA basis plus those added at each previous iteration, i.e., the reference set for unit k is not allowed to change to a basis of units (facet) farther from k.

Assuming that all surplus variables are zero at the Nth iteration, the relations at (3.24) can be "telescoped," surplus free, by first replacing the surplus value $s_{rk}^{(N-1)*}$ in iteration N-1 with the left hand side of its surplus free successor, iteration N, and continuing these successor-to-predecessor substitutions until the following single relation is obtained in the DEA iteration:

$$\begin{aligned}
&\sum_{j \varepsilon E_k^{(1)}} \lambda_j^* y_{rj} + (-1)^1 \sum_{j \varepsilon E_k^{(1)}} \lambda_j^{(1)*} y_{rj} + \cdots \\
&\quad + (-1)^N \sum_{j \varepsilon E_k^{(N)}} \lambda_j^{(N)*} y_{rj} + \cdots \\
&\quad + (-1)^1 \sum_{j \varepsilon \bar{E}_k^{(1)}} \gamma_j^{(1)*} y_{rj} + \cdots \\
&\quad + (-1)^N \sum_{j \varepsilon \bar{E}_k^{(N)}} \gamma_j^{(N)*} y_{rj} = y_{rk} \quad r = 1,2,\ldots,s
\end{aligned} \tag{3.2}$$

Similarly, the iterative optimal relations for dual input constraints can be telescoped to obtain:

$$\begin{aligned}
&- \sum_{j \varepsilon E_k^{(1)}} \lambda_j^* x_{ij} + (-1)^2 \sum_{j \varepsilon E_k^{(1)}} \lambda_j^{(1)*} x_{ij} + \cdots \\
&\quad + (-1)^{N+1} \sum_{j \varepsilon E_k^{(N)}} \lambda_j^{(N)*} x_{ij} + \cdots
\end{aligned}$$

$$+ (-1)^2 \sum_{j \varepsilon \overline{E}_k^{(1)}} \gamma_j^{(1)*} x_{ij} + \cdots \tag{3.27}$$

$$+ (-1)^{N+1} \sum_{j \varepsilon \overline{E}_k^{(1)}} \gamma_j^{(N)*} x_{ij} + \theta_k^* x_{ik}$$

$$- \left[\sum_{t=1}^{N} (-1)^t \omega_k^{(t)*} \right] x_{ik} = 0 \qquad i = 1,2,\ldots,m$$

By collecting terms, these lengthy equality relations can be reduced to:

$$\sum_{j=1}^{n} \lambda_j'^* y_{rj} = y_{rk} \tag{3.28}$$

$$r = 1,2,\ldots,s$$

$$-\sum_{j=1}^{n} \lambda_j'^* x_{ij} + \left[\theta_k^* - \sum_{t=1}^{N} (-1)^t \omega^{(t)*} \right] x_{ik} = 0$$

$$i = 1,2,\ldots,m$$

Where terms are collected as follows:

$$\lambda_j'^* = \sum_{t \varepsilon L_j} (-1)^t \lambda_j^{(t)*} + \sum_{t \varepsilon G_j} (-1)^t \gamma_j^{(t)*} \tag{3.29}$$

$\lambda_j'^* = 0$ if unit j is not related to a basic dual variable in any iteration

$$L_j = \{t \mid \lambda_j^{(t)} \text{ is basic}\},\ t = 0,1,2,\ldots,N$$

$$G_j = \{t \mid \nu_j^{(t)} \text{ is basic}\},\ t = 1,2,\ldots,N$$

$$\lambda_j^{(0)*} = \lambda_j^*$$

The expression

$$\omega_k^* = \theta_k^* - \sum_{t=1}^{N} (-1)^t \omega_k^{(t)*} \tag{3.30}$$

represents the "intensity" of the reduction of unit k's inputs when unit k is compared to s+m-1 actual efficient observations from the nearest complete frontier facet. The above equation (3.30) for ω_k^* illustrates how the DEA intensity θ_k^* is reduced by the iterative second stage process of bringing nearby efficient observations into the basis as a replacement for surplus vectors. This ω_k^* value is the desired minimum efficiency measure.

Using the relations at (3.28) and the optimal primal values of iteration N ($\mu_{rk}^{(N)*}$, r = 1,2,...,s and $\nu_{ik}^{(N)*}$, i = 1,2,...,m) one can develop an alternate expression for ω_k^* which provides further proof that it is the minimum efficiency measure. The equations at (3.28) can be multiplied by the appropriate primal value to obtain:

$$\mu_{rk}^{(N)*}\left[\sum_{j=1}^{n} \lambda_j'^{*} y_{rj}\right] = \mu_{rk}^{(N)*} y_{rk} \qquad r = 1,2,\ldots,s$$

(3.31)

$$-\nu_{ik}^{(N)*}\left[\sum_{j=1}^{n} \lambda_j'^{*} x_{ij}\right] = -\nu_{ik}^{(N)*}(\omega_k^{*} x_{ik}) \qquad i = 1,2,\ldots,m$$

Summing all of these equations, one obtains:

$$\sum_{j=1}^{n}\left[\lambda_j'^{*}\left[\sum_{r=1}^{s} \mu_{rk}^{(N)*} y_{rj} - \sum_{i=1}^{m} \nu_{ik}^{(N)*} x_{ij}\right]\right] \qquad (3.32)$$

$$= \sum_{r=1}^{s} \mu_{rk}^{(N)*} y_{rk} - \omega_k^{*} \sum_{i=1}^{m} \nu_{ik}^{(N)*} x_{ik}$$

The part of equation (3.32) in the inner brackets is zero for every unit j in the basis at iteration N, and $\lambda_j'^{*}$ is zero for every unit j not in the basis at iteration N. Thus, the left hand side of (3.32) is zero. Furthermore,

$$\sum_{i=1}^{m} \nu_{ik}^{(N)*} x_{ik} = 1 \qquad (3.33)$$

as required by the primal linearizing constraint. Thus, equation (3.32) can be rewritten:

$$0 = \sum_{r=1}^{s} \mu_{rk}^{(N)*} y_{rk} - \omega_k^{*} \sum_{i=1}^{m} \nu_{ik}^{(N)*} x_{ik} \qquad (3.34)$$

$$= \sum_{r=1}^{s} \mu_{rk}^{(N)*} y_{rk} - \omega_k^{*}(1)$$

or

$$\omega_k^* = \sum_{r=1}^{s} \mu_{rk}^{(N)*} y_{rk}$$

Note, the primal constraints in (3.22) and the equality at (3.34) guarantee:

$$\sum_{r=1}^{s} \mu_{rk}^{(N)*} y_{rj} - \sum_{i=1}^{m} \nu_{ik}^{(N)*} x_{ij} = 0 \quad j \in E_k^{(N)} \qquad (3.35)$$

$$\sum_{r=1}^{s} \mu_{rk}^{(N)*} y_{rk} - \sum_{i=1}^{m} \nu_{ik}^{(N)*} (\omega_k^* x_{ik}) = 0$$

$$\sum_{r=1}^{s} \mu_{rk}^{(N)*} \left(\frac{y_{rk}}{\omega_k^*}\right) - \sum_{i=1}^{m} \nu_{ik}^{(N)*} x_{ik} = 0$$

which means the vectors

$$\begin{pmatrix} Y_k \\ \\ \omega_k^* X_k \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} \frac{1}{\omega_k^*} \cdot Y_k \\ \\ X_k \end{pmatrix}$$

would be rated efficient by DEA, i.e., both are frontier points which can be expressed as a linear combination of nearby facet vectors.

The scaled vector of inputs $\omega_k^* X_k$, contains the reduced amounts which would exist at an extended frontier

point having the same proportionate mix of inputs and the same outputs as unit k. The scaled vector of outputs, $\frac{1}{\omega_k^*} \cdot Y_k$ contains the increased amounts which would exist at an extended frontier point having the same proportionate mix of outputs and the same inputs as unit k.

The essential characteristics of the minimum efficiency measure developed above can be restated in summary fashion as follows.

Property 2. If the second stage evaluation terminates after N iterations with all surplus variables nonbasic, then the minimum efficiency measure is

$$\omega_k^* = \sum_{r=1}^{s} \mu_{rk}^{(N)*} y_{rk} \qquad (3.36)$$

and the vectors

$$\begin{pmatrix} Y_k \\ \omega_k^* X_k \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} \frac{1}{\omega_k^*} Y_k \\ X_k \end{pmatrix}$$

are points on the extended frontier and can be used as estimates of "values if efficient."

Facet Information from Second Stage Model

Thus far in this section the focus has been on the minimum efficiency concept and little has been said about the identification of additional basis units and associated multipliers which provide valuable information about the frontier neighborhood. The facet that is generated during this second stage process can be defined as the set of all possible input/output vectors which can be expressed as convex linear combinations of observed input/output vectors in the optimal basis.

In mathematical terms, the reference facet for unit k is:

$$F_k = \{(Y,X) \mid (Y,X)^T = \sum_{j \in E_k^{(N)}} \lambda_j P_j \text{ and } \sum_{j \in E_k^{(N)}} \lambda_j = 1\}$$

where

$$(Y,X) = (y_1, y_2, \ldots, y_s, x_1, x_2, \ldots, x_m)$$

and

$$P_j^T = (y_{1j}, y_{2j}, \ldots, y_{sj}, x_{1j}, x_{2j}, \ldots, x_{mj}), \quad j \in E_k^{(N)}$$

is a member of the set of observed basis vectors. If the "values if efficient" vectors are also thought to represent feasible production possibilities, then these vectors could be included in the generation of convex linear combinations thereby accepting points on the extended frontier as production possibilities.

The primal constraints of the model for vectors in the basis guarantee that within this facet the rates of substitution and the marginal productivities are:

<u>Rates of Substitution</u>:

$$\frac{\partial x_p}{\partial x_q} = -\frac{\nu_{qk}^{(N)*}}{\nu_{pk}^{(N)*}} \quad q \neq p \quad q,p \in \{1,2,\ldots,m\}$$

$$\frac{\partial y_p}{\partial y_q} = -\frac{\mu_{qk}^{(N)*}}{\mu_{pk}^{(N)*}} \quad q \neq p \quad q,p \in \{1,2,\ldots,s\} \tag{3.38}$$

<u>Marginal Productivities</u>:

$$\frac{\partial y_r}{\partial x_i} = \frac{\nu_{ik}^{(N)*}}{\mu_{rk}^{(N)*}} \qquad i = 1,2,\ldots,m; \; r = 1,2,\ldots,s \tag{3.39}$$

Note, if the denominator of any of the above ratios is zero, then the ratio is not defined and the facet is not proper.

The second stage model requires that each multiplier be ≥ 0, and there is a possibility that one

or more of the multipliers could be zero at the final iteration, which implies that the frontier facet is assumed to be parallel to each of the input or output axes associated with the zero multipliers. For example, if $\nu_{pk}^{(N)*} = 0$ then

$$\frac{\partial x_q}{\partial x_p} = -\frac{\nu_{pk}^{(N)*}}{\nu_{qk}^{(N)*}} = 0 \text{ for every } q \text{ such that } \nu_{qk}^{(N)*} > 0 \tag{3.40}$$

which means that the change in x_q along the facet in a direction parallel to the x_p axis is assumed to be zero for every input q having a positive multiplier. Of course, the ratio is not defined if the denominator is zero.

A multiplier for input p would be assigned a zero value by the model if the rates of substitution of that input with others in the facet tended to be nonnegative or if the marginal productivities tended to be nonpositive, conditions which would occur if the input p did not trade off with other inputs or if there existed a zero or negative correlation between input p and the outputs (a syphoning effect).

Property 3. At the final iteration of the second stage method, the appearance of zero multipliers

in the optimal basis indicates that the facet formed from the basis vectors is not proper because it fails to achieve negative rates of substitution and positive marginal productivities.

Test Case

Clark [23] performed experiments to determine if the model would behave as expected in producing the desired frontier information. The Educational Productivity Council 1980-1981 data base provided the set of input and output observations used in the experiments. One inefficient school, Decision Making Unit (DMU) 61, was selected for evaluation, and all 103 efficient schools were used in the frontier reference set.

The inefficient DMU 61 ($h = .841$) was chosen for the following reasons: (1) it was not fully enveloped, i.e., eight of the thirteen inputs and outputs had positive slack amounts in the DEA optimal solution which caused eight multipliers to assume the lower bound value $\epsilon = 10^{-6}$; and (2), only four efficient schools were identified as being members of unit 61's facet. The results of the DEA evaluation are summarized in Table 3.1.

TABLE 3.1

DEA Analysis for Decision Making Unit 61

Summary of Results

Efficiency = .841

	Value Measured	Slack	Multipliers
Output 1	71.4000	5.4544	.000001
Output 2	78.1300	0	.010759
Output 3	72.0000	7.2676	.000001
Input 1	50.0000	0	.003246
Input 2	90.0000	8.1283	.000001
Input 3	20.0000	0	.007437
Input 4	21.9100	4.3631	.000001
Input 5	5.2100	1.2510	.000001
Input 6	90.5700	0	.006595
Input 7	100.0000	.9662	.000001
Input 8	100.0000	18.7373	.000001
Input 9	92.8600	5.3923	.000001
Input 10	85.7100	0	.001064

Units Defining Frontier Facet: DMUs 12, 13, 68 and 98

Appendix 1 contains the details of the optimal DEA solution for DMU 61 and the identification numbers for frontier reference units (constraint numbers).

The second stage method was tried using the EPC data set and then compared to an alternative procedure for generating the nearest facet. The alternative method involved iterative subtraction of slack input amounts from the observed input values of DMU 61 and addition of output slack to the output observations, a process which produces adjusted input and output amounts that move DMU 61 closer to full envelopment at each iteration. When DEA is retried with these new amounts, conditions become more favorable for the entry into the basis of other variables associated with nearby frontier observations. This process guarantees that the units identified are members of the nearest DEA facet. Unfortunately, 17 iterations were required to complete the evaluation of DMU 61 using the alternate method, probably a result of rounding error, and only six additional basis units could be identified for a total of ten with two of the full set of s+m-1 not identified.

A comparison of the results of the Charnes, Cooper and Rhodes DEA model and those obtained from the

alternate and second stage methods is given in Tables 3.2 and 3.3.

As expected, the second stage evaluation identified more facet units, provided more multipliers greater than ϵ and had a lower efficiency measure than either DEA or the alternate method. Furthermore, the second stage method required only three iterations in contrast to the 17 required by the alternate technique.

In all three cases the multiplier of Input 5 remained at its lower bound of ϵ and the second stage evaluation revealed that this input would frustrate any further attempts to obtain a complete basis of 12 units. A subsequent analysis revealed that Input 5 is negatively correlated with Output 3 within the facet of eleven units formed by the second stage method. For this reason, its multiplier could not enter the basis at an amount greater than ϵ without first driving some other multiplier from the basis.

At this point, the second stage process works as expected. In the next chapter, this technique will be used again in assessing the relative efficiency of Air Force wings and in describing the characteristics of neighborhood efficiency frontiers.

TABLE 3.2

Comparison of Multipliers and Efficient Measures

	DEA	Alternate Method	Second Stage
Output 1	.000001(ϵ)	.0003596	.0001803
Output 2	.010759	.0104064	.0096060
Output 3	.000001(ϵ)	.000001(ϵ)	$\boxed{.0006735}$
Input 1	.003246	.0032422	.0031641
Input 2	.000001(ϵ)	.0000952	.001066
Input 3	.007437	.0078976	.0074301
Input 4	.000001(ϵ)	.0001447	.0002495
Input 5	.000001(ϵ)	.000001(ϵ)	.000001(ϵ)
Input 6	.006595	.0055419	.0054183
Input 7	.000001(ϵ)	.0003076	.0003101
Input 8	.000001(ϵ)	.0010049	.0011169
Input 9	.000001(ϵ)	.0002436	.0002172
Input 10	.001064	.0004477	.0002862
Efficiency	.841	.839	$\boxed{.811}$

TABLE 3.3

Comparison of Facet DMUs

(DMU # = Constraint # in Appendix 1)

DEA		Alternate Method	Second Stage
1.	12	12	12
2.	13	13	13
3.	--	20	20
4.	--	36	36
5.	--	63	63
6.	68	68	68
7.	--	73	73
8.	--	79	79
9.	--	--	$\boxed{84}$
10.	--	89	89
11.	98	98	98
12.	--	--	--

CHAPTER IV

USING THE DEA MODEL AND EXTENSIONS IN EVALUATING AIR FORCE WINGS

Introduction

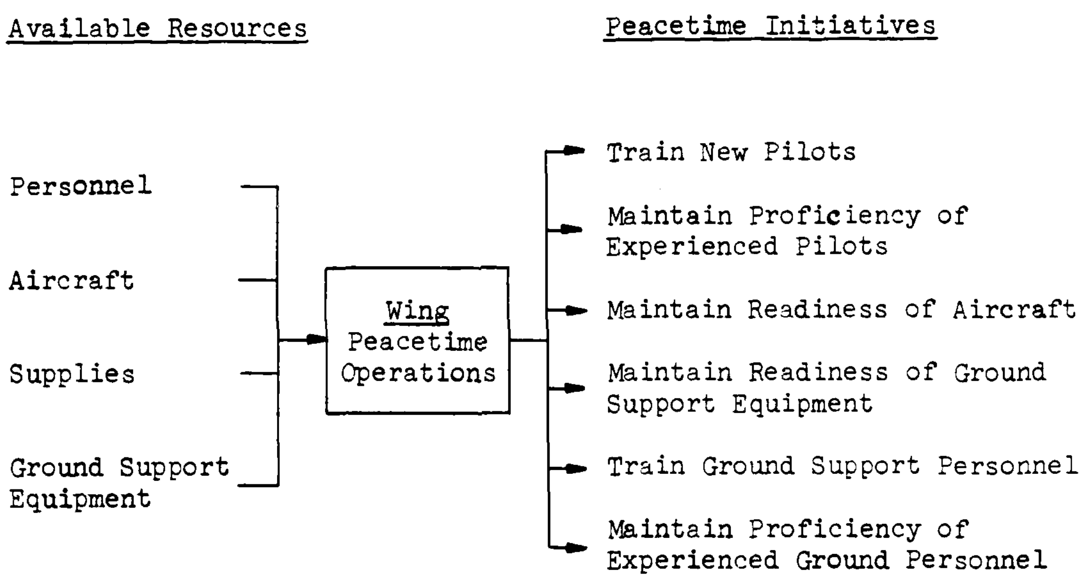
Air force tactical fighter wings are expected to maintain high levels of combat readiness of aircrews, fighter aircraft and ground support resources. The Data Envelopment Analysis (DEA) technique with extensions developed in this study appear to have significant potential for use by the Air Force in monitoring the efficiency of operations and planning courses of action which will remedy problems and increase the capability of combat units.

The primary purpose of this chapter is to present a small numerical example which provides insight into the complexity of the wing evaluation problem and which illustrates the use of DEA and the extensions developed in Chapter III. The input and output measures used in this chapter are similar to those used by Air Force commanders and resource managers, and were chosen to highlight the key objectives, operating characteristics and input factors of wings. Some of the data used are fictitious but it is

hoped reasonable, and were generated for purely illustrative purposes.

Selection of Input and Output Measures and Data Used

The input and output measures used in this analysis will take into account, either directly or indirectly, the following resources and peacetime initiatives of wings:



Data were generated for fourteen fictitious tactical fighter wings, eight of which (A through H) are assumed to be organized under one intermediate headquarters

and the remaining six (I through N) under another, with both intermediate headquarters reporting to the Tactical Air Command Headquarters. Wings are assumed to fall into one of three mission categories: combat operations, aircraft familiarization (training) or both. Furthermore, each wing is assumed to have one assigned aircraft type, and these different types can be further classified in terms of age and complexity. These classifications are shown in Table 4.1.

The particular input and output measures selected for use in this example are defined as follows:

Outputs Selected

Output 1: Net Combat-Practice Sorties Flown. A single sortie involves the departure, flight and full stop landing (not touch-and-go) of one fighter aircraft. When the aircraft lands, ground operations commence to return the aircraft to mission capable status and prepare for the next sortie. Figure 4.1 shows the typical activities occurring during sortie generation and recovery operations.

The number of sorties flown can be viewed as a surrogate measure of wing output related to the training

Table 4.1

Wing Classifications

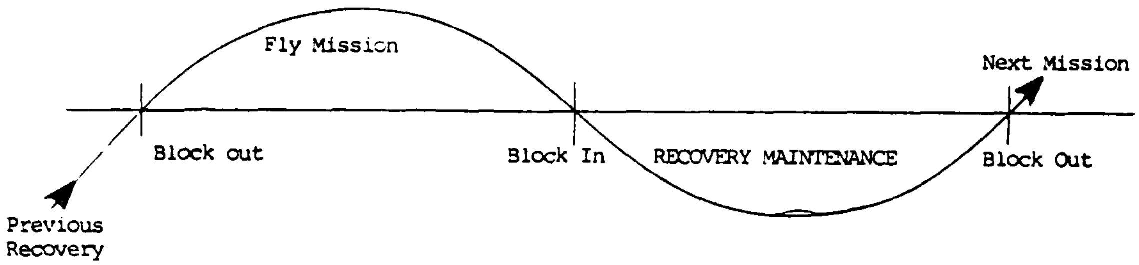
Intermediate Headquarters	Wings	Wing Missions	Aircraft Descriptions Type	Age	Complexity
I	A	Ops*	1	New	Complex
I	B	Ops	1	New	Complex
I	C	Ops	2	Old	Complex
I	D	Ops	2	Old	Complex
I	E	Tng**	3	Old	Complex
I	F	Tng	4	Very Old	Complex
I	G	Ops + Tng	5	New	Complex
I	H	Ops	6	New	Simple+
II	I	Ops	1	New	Complex
II	J	Tng	1	New	Complex
II	K	Tng	2	Old	Complex
II	L	Ops + Tng	7	Old	Very Complex++
II	M	Ops + Tng	7	Old	Very Complex
II	N	Tng	8	Fairly New	Simple

* Ops : Combat Operations

** Tng : Aircraft Familiarization Training

+ Simple : easy to troubleshoot and fix

++ Very complex : very difficult to troubleshoot and fix



Block Out	Block In
Taxi	Secure Aircraft
End of Runway checks	Debrief Crew
(Possible Ground Abort)	Refuel
Launch	Service with oil
Fly	Inspect
(Possible Air Abort)	Fix If Required
Complete Mission	Reconfigure tanks, racks adaptors (if required)
Return to Base	Load Munitions
End of Runway Checks	Combat Ready
Taxi and Park	Pilot Arrival & Inspection
Block In	Start Aircraft
	Block Out

Figure 4.1

Typical Sortie Cycle Activities

of aircrews and the exercising of ground support functions to maintain high levels of personnel readiness and to keep mission essential equipment in good operating condition. Controllers and analysts at the wing monitor the mission departures and arrivals of aircraft, and they keep cumulative sortie records by day, week, month and year.

One category of sorties, here labeled "net combat-practice," is defined as those sorties flown by fully qualified pilots to maintain proficiency in combat tactics. This category excludes sorties flown in training new pilots and those resulting in aborts, e.g.,

Annual Total Sorties Attempted	10,000
Annual Training Sorties	- 2,500
Annual Air Aborts	- 100
Annual Ground Aborts	- 400
Annual Net Combat-Practice Sorties	7,000

The data used in computing Output 1 for each of the 14 wings (A,B,C,...,N) in this example are shown in the last column of Table 4.2.

Output 2: Flight Training Sorties. This measures the degree to which a wing is active in training pilots. The annual requirements for training sorties are established by operations and tracked by analysts. Annual

Table 4.2

Net Combat Practice Sorties

Wings	1 Total Attempted (Annually)	2 Air Aborts (Annually)	3 Ground Aborts (Annually)	4 Training Sorties (Annually)	5 Net Combat Practice Sorties (Annually)*
A	15,876	240	444	0	15,192
B	11,095	300	360	0	10,435
C	14,975	384	600	0	13,991
D	12,888	240	300	0	12,348
E	18,117	384	540	17,193	0
F	10,101	120	240	9,741	0
G	13,347	288	480	9,148	3,431
H	6,961	96	192	0	6,673
I	16,646	312	324	0	16,010
J	20,477	240	576	19,661	0
K	5,007	125	242	4,640	0
L	8,015	168	315	5,021	2,511
M	9,105	96	180	6,147	2,682
N	34,998	720	1020	33,257	0

*Column 5 = (Column 1) - (Column 2) - (Column 3) - (Column 4)

training sorties for each of the hypothetical wings in this example are shown in the second to last column of Table 4.2

<u>Output 3: Mission Capable Aircraft Days</u>. An aircraft can be Not Mission Capable for Supply reasons only (NMCS), for maintenance reasons only (NMCM) or both (NMCB). Thus, let there be j = 1,2,...,n aircraft. The percent of time that the jth aircraft is mission capable during the year is:

$$\%MC_j = 100 - (\%NMCM_j) - (\%NMCS_j) - (\%NMCB_j) \tag{4.1}$$

Let T_j be the total number of days the jth aircraft is on hand at the unit and let $T = \sum_j T_j$ be the total available aircraft days at the unit. Then the total number of annual Mission Capable Aircraft Days (MCAD) is:

$$\begin{aligned} MCAD &= \sum_j MCAD_j = \sum_j T_j[\%MC_j] \qquad (4.2) \\ &= \sum_j T_j[100 - (\%NMCM_j) - (\%NMCS_j) - (\%NMCB_j)] \end{aligned}$$

Each wing or squadron is expected to maximize the number of mission capable aircraft available at any point in the time to remain prepared for war. Controllers monitor the mission capable status of aircraft on a continuing basis.

The data used in computing output 3 for each wing in this example are shown in Table 4.3.

Inputs Selected

Input 1: Average Available Aircraft. The average number of aircraft on hand during the period can be computed by dividing the sum of not mission capable days and mission capable days by the number of days in the period, e.g., using the annual data in Table 4.3 for wing A:

Not Mission Capable Aircraft Days	10,486	(column 2)
Mission Capable Aircraft Days	+ 15,794	(column 3)
Total Aircraft Days	26,280	(column 1)
Days in Period	÷ 365	
Average Daily Aircraft Available	72	

Values for each wing are shown in Table 4.4, column 4.

Input 2: Supply Support Factor. Two important considerations in assessing supply support of wing flight operations are: (1) Were mission essential parts available and provided upon request? (2) If mission essential parts are not available, how long did mechanics have to wait for these parts? The fewer parts that are available

Table 4.3

Mission Capable Aircraft Days

Wings	Total Available Aircraft Days (T) (Annually)	Total Not Mission Capable Aircraft Days (Annually)	Total Mission Capable Aircraft Days (MCAD) (Annually)
A	26,280	10,486	15,794
B	16,316	6,233	10,083
C	25,222	10,670	14,522
D	18,761	4,990	13,771
E	30,733	9,066	21,667
F	19,126	6,331	12,795
G	23,433	6,585	16,848
H	11,863	1,685	10,178
I	26,207	10,011	16,196
J	35,734	13,437	22,297
K	8,103	3,541	4,562
L	25,514	14,697	10,817
M	28,981	15,968	13,012
N	40,880	11,120	29,760

Table 4.4

DEA Output and Input Data

	Outputs			Inputs			
	1	2	3	4	5	6	7
Wings	Net Combat Practice Sorties (Annually)	Flight Training Sorties (Annually)	Mission Capable Aircraft Days (Annually)	Daily Average Available Aircraft (During the Year)	Supply Support Factor (Annual Average)	Available Labor Hours (Annually) (x 1000)	Mission Essential Equipment Availability (Days During the Year)
A	15,192	0*	15,794	72	6.1	1,980	81,000
B	10,435	0	10,083	45	17.3	1,408	55,000
C	13,991	0	14,552	69	26	1,936	80,625
D	12,348	0	13,771	51	13	1,496	55,375
E	0	17,193	21,667	84	17.3	2,508	100,000
F	0	9,741	12,795	52	10.4	1,320	57,500
G	3,431	9,148	16,848	64	25.9	1,302	75,000
H	6,673	0	10,178	33	26	924	37,125
I	16,010	0	16,196	72	13	1,980	79,800
J	0	19,661	22,297	98	8	2,640	110,250
K	0	4,640	4,562	22	103.7	740	26,750
L	2,511	5,021	10,817	70	25.9	1,188	83,400
M	2,682	6,147	13,012	80	6.9	1,179	90,000
N	0	33,257	29,760	112	34.5	4,400	126,000

*DEA will not allow zero amounts in inputs or outputs. Thus, relative small values between one and 10 were substituted for zero in several DEA trials. The same results were obtained in each trial implying that any amount less than ten is sufficiently small relative to the observed positive sortie amounts and can be used as an acceptable approximation of zero.

or the longer one has to wait, the lower the supply support. One might therefore construct a measure as follows.

Suppose there are $j = 1,2,\ldots,n$ mission essential parts. Let D_j = the demand for the jth mission essential part during the year being considered. Let R_j be the average length of time from request to receipt of the jth part. Then the weighted (weighted by demand) average number of hours awaiting delivery of a single mission essential part would be:

$$(\sum_j D_j R_j) \div (\sum_j D_j) \qquad (4.3)$$

This measures supply <u>non</u>support; thus, the measure of supply support should have a reciprocal relation to this sum, perhaps $(\sum_j D_j / \sum_j D_j R_j) \times P$ where P is a scalar multiplier to enlarge each of the values to a size appropriate for DEA. Supply support factors for this example were arbitrarily assigned as shown in Table 4.5.

<u>Input 3: Available Labor Hours (In Thousands of Hours)</u>. This measures the size of the available workforce which generally varies proportionately with the levels of flying and ground support activities at each wing. See Table 4.4 for the values used in the example.

Table 4.5

Supply Support Factor

Wings	Average Number of Hours Awaiting Delivery of a Single Mission Essential Part (Annual Average) (h)	Supply Support Factor* $\left(\frac{1}{h} \times 100\right)$
A	16.39	6.1
B	5.78	17.3
C	3.85	26.0
D	7.69	13.0
E	5.78	17.3
F	9.62	10.4
G	3,86	25.9
H	3.85	26.0
I	7.69	13.0
J	12.50	8.0
K	.96	103.7
L	3.86	25.9
M	14.49	6.9
N	2.90	34.5

* Signifies the assumed reciprocal relationship between time awaiting delivery of parts and the two outputs, sorties flown and mission capable aircraft days.

Input 4: Mission Essential Equipment Availability. Ground equipment authorizations are determined at management levels above the wing, but wing level managers have some control over the proportion of assigned equipment which is serviceable at any one time. Higher levels of availability and serviceability of wing mission essential equipment should provide smoother more efficient flying and maintenance operations resulting in greater output. Levels of ground equipment authorizations also vary proportionately with levels of flying and required ground support activities, but wings seldom have equipment levels equal to authorizations. One measure might be $\sum_j A_j$ where A_j is the amount of time in days that the jth piece of mission essential equipment is assigned to the wing. This measure does not reflect the difference in value of individual equipment types, e.g., a power cart used in starting aircraft might be more valuable to the operations than a tow bar. See Table 4.4 for the arbitrarily assigned values used in this example to represent cumulative days of mission essential equipment availability.

Computations and Interpretations Using the Charnes, Cooper and Rhodes DEA Model

The Charnes, Cooper and Rhodes DEA relative efficiency of each wing was computed using the data in Table 4.4 and the BDEAV5 code developed by Elam [26]. The BDEAV5 code is a modified version of DEA3 which was developed by Ali, Bessent, Bessent and Kennington [3]. Elam's revision provides for interactive selection of inputs, outputs and reference units and adds the capability to list the units defining a particular local frontier along with the associated values for λ variables in the optimal basis.

Results from this trial of the Charnes, Cooper and Rhodes DEA method are summarized below in two tables. Data for the wings classified as efficient by the model are presented in Table 4.6, and results for those classified as inefficient are shown in Table 4.7.

In Table 4.6, the units classified as efficient by the model are listed with their observed values and the DEA optimal values of multipliers. Any efficient unit, say D, serves as its own frontier reference point in the Charnes, Cooper and Rhodes DEA evaluation; i.e., its lambda value, λ_D^*, equals one, the other lambdas have a value

Table 4.6

DEA Observed Values and Multipliers for Efficient Wings (h_k = 1.0)

	1 Efficient Wings	2 Net Combat Practice Sorties (Annually)	3 Training Sorties (Annually)	4 Mission Capable Aircraft Days (Annually)	5 Daily Average Available Aircraft (During the year)	6 Supply Support Factor (Annual Average)	7 Available Labor Hours (Annually) (x 1000)	8 Mission Essential Equipment Availability (Days During the Year)
Observed Values	A	15,192.0	1.0*	15,794.0	72.0	6.1	1,980.0	81,000.0
	D	12,348.0	1.0*	13,771.0	51.0	13.0	1,496.0	55,375.0
	E	1.0*	17,193.0	21,667.0	84.0	17.3	2,508.0	100,000.0
	F	1.0*	9,741.0	12,795.0	52.0	10.4	1,320.0	57,500.0
	G	3,431.0	9,148.0	16,848.0	64.0	25.9	1,302.0	75,000.0
	H	6,673.0	1.0*	10,178.0	33.0	26.0	924.0	37,125.0
	I	16,010.0	1.0*	16,196.0	72.0	13.0	1,980.0	79,800.0
	J	1.0*	19,661.0	22,297.0	98.0	8.0	2,640.0	110,250.0
	M	2,682.0	6,147.0	13,012.0	80.0	6.9	1,179.0	90,000.0
	N	1.0*	33,257.0	29,760.0	112.0	34.5	4,400.0	126,000.0
Multipliers	A	.000065	.000055	ε	.011688	.012375	ε	ε
	D	.000080	ε	ε	ε	ε	ε	.000018
	E	ε	.000003	.000044	.009367	.006394	ε	ε
	F	ε	.000007	.000073	ε	.013319	.000038	.000014
	G	ε	.000006	.000056	.011568	.003503	.000072	ε
	H	.000042	ε	.000071	ε	ε	ε	.000027
	I	.000061	ε	ε	.000581	.007517	.000394	ε
	J	ε	.000003	.000042	.005516	.008462	ε	.000004
	M	.000005	ε	.000075	.003768	.019255	.000404	ε
	N	ε	.000005	.000028	ε	ε	ε	.000008

* For DEA it is necessary to use the relatively small value of 1.0 because zero amounts are not allowed.

of zero at optimality and $\theta_D^* = h_D^* = 1$. Furthermore, in the optimal solution of the DEA dual model, all slack values s_{ik}^{-*} and s_{rk}^{+*} are zero when the unit k is efficient. As a result of these conditions, for frontier units the Charnes, Cooper and Rhodes values if efficient ($\hat{y}_{rk}$ and $\hat{x}_{ik}$) equal the observed values, i.e.,

$$\hat{y}_{rk} = y_{rk} + s_{rk}^{+*} = y_{rk} + 0 = y_{rk}, \quad r=1,2,\ldots,s \tag{4.4}$$

$$\hat{x}_{ik} = \theta_k^* x_{ik} - s_{ik} = (1)x_{ik} - 0 = x_{ik}, \quad i = 1,2,\ldots,m$$

The primary areas of focus for the discussion in this section are the identification of efficient wings, the patterns of multipliers, and the evaluation of inefficiencies using the Charnes, Cooper and Rhodes DEA model. Inefficient wings are evaluated through comparison with frontier facets containing one or more of the efficient wings in Table 4.6. How this is done will be described later when the results for inefficient wings are reviewed. There are, however, some interesting observations that can be made regarding the μ_{rk}^* and v_{ik}^* multipliers of efficient wings.

Note first that nearly one half of the multipliers in Table 4.6 are ϵ values. The appearance of epsilons in the optimal DEA solution for a particular wing

implies that the wing achieved the maximum efficiency measure by avoiding actual vectors of observations which if taken into account would reduce the efficiency rating, i.e., an optimal dual basis was formed which includes slack variables. At optimality, the multipliers provide the highest possible efficiency rating for the wing in question; and, if no alternative optima exist, any other feasible assignment of multiplier values would reduce the efficiency measure. For example, if additional multipliers of wing D are forced to exceed their lower bound, slack variables would leave the dual basis causing λs to enter (provided such entry is feasible) and the efficiency measure would decrease, unless of course alternative optima are available.

Each of the wings shown in Table 4.6 achieved its efficient rating by having a combination of slack and lambda variables in its optimal dual basis. Of course, all the s_{rk}^{+*} and s_{ik}^{-*} slack values for each wing were zero; otherwise, the wing could not be classified as efficient. If an optimal dual basis of an efficient wing had occurred with no slack variables in it (or a primal with no ϵ multiplie .), then a proper facet would have been known to exist. Unfortunately, a basis free of slack variables

did not occur, and there is no guarantee that a proper facet exists.

Note also that the pattern of multipliers with a value of epsilon in the first two columns conforms closely to the pattern of observed output values which were arbitrarily assigned a relatively small value of 1.0 as a substitute for zero. This is understandable because the Charnes, Cooper and Rhodes DEA model, in maximizing the efficiency measure, would ordinarily prefer to avoid assigning relatively large multipliers to such obviously small output amounts. However, since the model considers all observations simultaneously, one cannot always predict which observed values will receive an epsilon multiplier. All input and output observations of a wing are assessed relative to the observed values of other wings; and, as stated previously, the combination of multipliers assigned at optimality will produce the highest possible efficiency rating.

Note that the wing A multiplier for training sorties was given a value of $.000055 > \epsilon$ by the model in spite of the fact that the relatively small value 1.0 was used as a measure of the output called training sorties. Any other feasible assignment of multipliers for wing A

would produce an efficiency rating less than one except when alternative optima are available.

In general, multipliers are given values equal to ϵ by the model based on a combination of factors including the relative size of the wing and its performance in producing outputs and conserving inputs. Epsilon multipliers are normally given to outputs which are too small relative to the observed amounts of other wings, or to inputs which are too large.

The input and output observations of wings in Table 4.6 form a frontier of relative efficiency which is used as a reference in evaluating the inefficiencies of the remaining wings. Each of the wings in Table 4.7 received a Charnes, Cooper and Rhodes DEA efficiency rating less than one (column 2) when compared to the subset of neighborhood frontier reference units shown in column 3. As expected, inefficient combat wings were compared to facets containing similar but efficient combat wings; the training wing K was compared to other wings with training missions; and wing L, a training and combat wing, was compared to an appropriate combination.

In every case in Table 4.7, the subset of efficiency frontier reference units contained less than the

TABLE 4.7

DEA Results for Inefficient Wings

1 Wings	2 DEA Efficiency (h_k^*)	3 Efficiency Frontier Units Defining Facet	4 Output and Input Names	5 Observed Values	6 u^* & v^* Multiplier Values	7 Intensity Adjusted Inputs ($h_k^* \cdot x_{ik}$)	8 Slack	9 CCR[1] Estimated Values If Efficient
B (Combat Wing)	.95	D (Combat Wing, $\lambda_D^* = .84508$)	NET COMBAT-PRACTICE SORTIES	10,435.0	.00009		0.0	10,435.0
			TRAINING SORTIES	0.0	.000074		0.0	0.0
			MISSION CAPABLE AIRCRAFT DAYS	10,083.0	ϵ		1,555.9	11,638.9
		N (Training Wing, $\lambda_N^* = .00005$)	AVAILABLE AIRCRAFT	45.0	.020968	42.8	0.0	42.8
			SUPPLY SUPPORT	17.3	ϵ	16.4	5.6	10.8
			LABOR HOURS (X 1000)	1,408.0	ϵ	1,337.6	84.2	1,253.4
			AVAILABLE EQUIPMENT	55,000.0	ϵ	52,250.0	5,880.8	46,369.2
C (Combat Wing)	.87	D (Combat Wing, $\lambda_D^* = 1.13306$)	NET COMBAT-PRACTICE SORTIES	13,991.0	.000061		0.0	13,991.0
			TRAINING SORTIES	0.0	ϵ		0.0	0.0
			MISSION CAPABLE AIRCRAFT DAYS	14,522.0	ϵ		1,051.3	15,603.3
			AVAILABLE AIRCRAFT	69.0	ϵ	60.0	2.6	57.4
			SUPPLY SUPPORT	26.0	ϵ	22.6	8.0	14.6
			LABOR HOURS (X 1000)	1,936.0	.000475	1,684.3	0.0	1,684.3
			AVAILABLE EQUIPMENT	80,625.0	ϵ	70,143.8	7,847.7	62,296.1
K (Training Wing)	.87	F (Training Wing, $\lambda_F^* = .09561$)	NET COMBAT-PRACTICE SORTIES	0.0	.000009		0.0	0.0
		G (Training and Combat Wing, $\lambda_G^* = .00231$)	TRAINING SORTIES	4,640.0	.000173		0.0	4,640.0
			MISSION CAPABLE AIRCRAFT DAYS	4,562.0	.000016		0.0	4,562.0
			AVAILABLE AIRCRAFT	22.0	ϵ	19.1	2.2	16.9
			SUPPLY SUPPORT	103.7	ϵ	90.2	86.0	4.2
		N (Training Wing, $\lambda_N^* = .11089$)	LABOR HOURS (X 1000)	740.0	.0001381	612.5	0.0	612.5
			AVAILABLE EQUIPMENT	26,750.0	ϵ	23,272.5	3,790.1	19,482.4
L (Combat and Training Wing)	.70	G (Training and Combat Wing, $\lambda_G^* = .60156$)	NET COMBAT-PRACTICE SORTIES	2,511.0	.000022		0.0	2,511.0
			TRAINING SORTIES	5,021.0	ϵ		482.8	5,503.8
			MISSION CAPABLE AIRCRAFT DAYS	10,817.0	.000059		0.0	10,817.0
			AVAILABLE AIRCRAFT	70.0	ϵ	49.0	9.1	39.9
			SUPPLY SUPPORT	25.9	ϵ	18.1	1.1	17.0
		H (Combat Wing, $\lambda_H^* = .06699$)	LABOR HOURS (X 1000)	1,188.0	.000771	831.6	0.0	831.6
			AVAILABLE EQUIPMENT	83,400.0	ϵ	58,380.0	11,725.9	46,654.1

[1] CCR means Charnes, Cooper, and Rhodes

$s+m-1 = 3+4-1 = 6$ units required for full envelopment. This lack of full envelopment is directly related to the presence of epsilon as optimal values for multipliers (column 6) and the positive s_{rk}^{+*} and s_{ik}^{-*} slack amounts shown in column 7. Thus, the Charnes, Cooper and Rhodes DEA efficiency ratings are overestimated, i.e., they are upper bound measures of efficiency.

According to Charnes, Cooper and Rhodes, each of the inefficient wings can achieve the frontier by adjusting all inputs and outputs according to the value if efficient formulae (refer to Table 4.7)

$$\hat{y}_{rk} = y_{rk} + s_{rk}^{+*}, \quad r=1,2,\ldots,s \qquad (4.5)$$

(column 9) = (column 5) + (column 8)

and

$$\hat{x}_{ik} = \quad = h_k^* x_{ik} - s_{ik}^{-*}, \quad i=1,2,\ldots,m$$

(column 9) = (column 7) - (column 8)

In other words, the frontier can be achieved by adding s_{rk}^{+*} output slack amounts to observed output values and subtracting s_{ik}^{-*} input slack amounts from intensity adjusted inputs. For example, the values if efficient for unit C in Table 4.7 were computed as follows:

Observed Outputs		Output Slack		Values if Efficient
13,991.0	+	0.0	=	13,991.0
0.0	+	0.0	=	0.0
14,552.0	+	1,051.3	=	15,603.3

(Observed Inputs) × Intensity		Input Slack		
69.0(.87)	-	2.6	=	57.4
26.0(.87)	-	8.0	=	14.6
1,936.0(.87)	-	0.0	=	1,684.3
80,625.0(.87)	-	7,847.7	=	62,296.1

The values if efficient were similarly computed for wings B, K, L, and all results have been included in Table 4.7.

Furthermore, the vector of values if efficient for any wing w can be expressed as a linear combination of wing w's efficiency frontier reference units. The following equality relation which appears in the dual DEA model illustrates this linear relation:

$$\sum_{j \varepsilon B^*} \lambda_j^* F_j = \begin{pmatrix} Y_w + S^{+*} \\ \theta_w^* X_w - S^{-*} \end{pmatrix} \tag{4.6}$$

$$\approx \begin{pmatrix} Y_w + S^{+*} \\ h_w^* X_w - S^{-*} \end{pmatrix}, \quad h_w^* \approx \theta_w^*$$

where B^* is the set of efficiency frontier reference units and

$$P_j = \begin{pmatrix} y_{1j} \\ y_{2j} \\ \vdots \\ y_{sj} \\ x_{1j} \\ x_{2j} \\ \vdots \\ x_{mj} \end{pmatrix} \quad Y_w = \begin{pmatrix} y_{1w} \\ y_{2w} \\ \vdots \\ y_{sw} \end{pmatrix} \quad h_w^* X_w = \begin{pmatrix} h_w^* x_{1w} \\ h_w^* x_{2w} \\ \vdots \\ h_w^* x_{mw} \end{pmatrix} \quad \text{and} \quad S^{+*} = \begin{pmatrix} s_{1w}^{+*} \\ s_{2w}^{+*} \\ \vdots \\ s_{sw}^{+*} \end{pmatrix} \quad S^{-*} = \begin{pmatrix} s_{1w}^{-*} \\ s_{2w}^{-*} \\ \vdots \\ s_{mw}^{+*} \end{pmatrix}$$

Thus, an alternate way of computing the value if efficient vector for unit C is:

$$\lambda_D \cdot \begin{pmatrix} y_{1D} \\ y_{2D} \\ y_{3D} \\ x_{1D} \\ x_{2D} \\ x_{3D} \\ X_{4D} \end{pmatrix} \approx \begin{pmatrix} y_{1C} + s_{1C}^{+*} \\ y_{2C} + s_{2C}^{+*} \\ y_{3C} + s_{3C}^{+*} \\ h_C^* x_{1C} - s_{1C}^{-*} \\ h_C^* x_{2C} - s_{2C}^{-*} \\ h_C^* x_{3C} - s_{3C}^{-*} \\ h_C^* x_{4C} - s_{4C}^{-*} \end{pmatrix} \quad (4.7)$$

or using numerical values from Tables 4.6 and 4.7,

$$
(1.133)\begin{pmatrix} 12,348 \\ 0 \\ 13,771 \\ 51 \\ 13 \\ 1,496 \\ 55,375 \end{pmatrix} = \begin{pmatrix} 13,990.3 \\ 0.0 \\ 15,602.5 \\ 57.8 \\ 14.7 \\ 1,695.0 \\ 62,740.0 \end{pmatrix} \simeq \begin{pmatrix} 13,991.0 + & 0.0 \\ 0.0 + & 0.0 \\ 14,552.0 + & 1,051.3 \\ (.87)69.0 - & 2.6 \\ (.87)26.0 - & 8.0 \\ (.87)1,936.0 - & 0.0 \\ (.87)80,625.0 - & 7,847.7 \end{pmatrix} = \begin{pmatrix} 13,991.0 \\ 0.0 \\ 15,603.3 \\ 57.4 \\ 14.6 \\ 1,684.3 \\ 62,296.1 \end{pmatrix}
$$

As implied by the appearance of epsilon values for multipliers and positive slack amounts in Table 4.7, and as suggested by the values if efficient, each inefficient wing should be able to achieve the same or greater outputs with fewer inputs, provided such reductions in inputs are feasible. It is unreasonable and unlikely that wing commanders would be willing to reduce the input amounts as suggested and in so doing give up the extra capability and strength these valued inputs might provide in combat. Instead, emphasis should be placed on making better use of existing resources to gain higher levels of output. With the aid of a second stage evaluation, analysts at headquarters and managers at wing level could use the input and output data from the related frontier units identified through second stage assessment and the

rates of substitution determined from multipliers to establish realistic goals for improving the outputs of inefficient wings. Headquarters might be particularly interested in using neighborhood frontier observations as a reference in finding alternate resource mixes which might enable combat wings B and C to achieve higher levels of mission capable aircraft, or in finding a mix which might enable training wing K to increase its sortie production to a level that has been demonstrated by other wings.

Furthermore, input and output "values if efficient" might become more useful for planning purposes if transformed into forms commonly used by analysts and managers involved in the Air Force planning process. Air Force reports and plans often use "rates" instead of the total output quantities used in this analysis. For example, one might transform values if efficient for sorties and mission capable days into the following rates which are commonly used in the Air Force:

1. Sortie Rate: Annual sorties flown by the wing, divided by the average number of aircraft assigned to the wing, divided by 12 months, to yield the average number of sorties flown by a single aircraft in one month.

2. Mission Capable Rate: The total number of mission capable aircraft, multiplied by 100 and divided by the total number of available aircraft days.

This suggested transformation for the two outputs of wing B are shown below:

$$\text{Sortie Rate Flown} = \frac{10{,}435}{45 \times 12} = 19.3$$

$$\text{Sortie Rate if Efficient} = \frac{10{,}435}{45 \times 12} = 19.3$$

$$\text{Mission Capable Rate Observed} = \frac{10{,}083 \times 100}{45 \times 365} = 61.4\%$$

$$\text{Mission Capable Rate if Efficient} = \frac{11{,}639 \times 100}{45 \times 365} = 70.9\%$$

Comparison of the above observed rates and the corresponding rates if efficient imply that wing B should continue its 19.3 sortie rate and increase its mission capable rate from 61.4 percent to 70.9 percent.

Surely there are other transformations and other methods of data analysis which could be used to take advantage of the wealth of information provided by the Charnes, Cooper and Rhodes DEA model and to exploit its capabilities in assimilating large sets of observations for evaluations of relative efficiency. This model is a step forward in organizing and assessing multiple factors

simultaneously. It can detect inefficiencies and provide information on resource averages or output shortages, information which if placed in the hands of knowledgeable and experienced managers could very well lead to worthwhile inquiries, explanations, and management action.

But other relevant information is needed to complete the efficiency evaluation of the not-fully-enveloped wings in Table 4.7. A second stage evaluation should be performed to determine the lower bound or minimum efficiency measure and to determine the marginal rates of substitution and productivity in the facets nearest to the inefficient wings.

Results of the Second Stage Evaluation

The second stage iterative procedure was performed for inefficient wings B, C, K and L to generate additional frontier information and to illustrate the use of the theoretical developments presented in Chapter III. In this section, the results of the second stage evaluation have been presented and compared to the DEA findings reported in the last section. Furthermore, the lower bound of efficiency of each wing (minimum efficiency ω_k^*) has been computed and compared to the upper bound

efficiency measure provided from solution of the Charnes, Cooper and Rhodes DEA model. Finally, the marginal rates of substitution and marginal rates of productivity relative to the facets derived from the second stage method have been reviewed and contrasted to those obtained via the Charnes, Cooper and Rhodes method.

Tables 4.8, 4.9, 4.10, 4.11 and 4.12 contain the results of the second stage evaluation and have been referred to throughout this discussion.

An Increase in the Number of Empirical Observations Defining the Facets

Table 4.8 shows the u^*_{rk} and v^*_{ik} multipliers produced by the Charnes, Cooper and Rhodes DEA trial together with the $\mu^{(N)*}_{rk}$ and $v^{(N)*}_{ik}$ multipliers obtained from the last iteration of the second stage analysis. Note, there are far fewer multiplier values at the ϵ lower bound in the second stage results of Table 4.8. This indicates that at the last iteration of the second stage model more frontier units were included in the comparison set or facet of each inefficient wing, which also means that more variables associated with vectors of empirical observations were present in the optimal basis.

Table 4.8

<u>Comparison of Charnes, Cooper and Rhodes DEA Multipliers and Those Obtained From Second Stage Evaluation</u>

	Optimal Multipliers For Output 1 (Combat-Practice Sorties)	Optimal Multipliers For Output 2 (Flight Training Sorties)	Optimal Multipliers For Output 3 (Mission Capable Aircraft Days)	Optimal Multipliers For Input 1 (Average Available Aircraft)	Optimal Multipliers For Input 2 (Supply Support Factor)	Optimal Multipliers For Input 3 (Available Labor Hours)	Optimal Multipliers For Input 4 (Mission Essential Equipment Availability)
Values From CCR* DEA:							
WING B	.0000900	.000074	ϵ	.020968	ϵ	ϵ	ϵ
WING C	.0000610	ϵ	ϵ	ϵ	ϵ	.000475	ϵ
WING K	.0000090	.0001730	.000016	ϵ	ϵ	.001381	ϵ
WING L	.0000220	ϵ	.0000590	ϵ	ϵ	.000771	ϵ
Values From Second Stage Evaluation:							
WING B	.0000319	.0000318	.0000381	ϵ	.0199245	ϵ	.0000119
WING C	.0000558	.0000575	.0000020	ϵ	.0069008	.0003839	.0000010
WING K (Proper Facet)	.0000146	.0000191	.0000423	.0074578	.0067169	.0001566	.0000011
WING L	ϵ	.0000047	.0000528	.0054335	.0036115	.0000761	.0000052

*Charnes, Cooper and Rhodes

Table 4.9 indicates the additional wing observations identified through second stage iterations included in the facets along with the associated lambda variables. The encircled values indicate the λ_j variables which were basic in the optimal solution of the Charnes, Cooper and Rhodes DEA model. For example, the original DEA optimal dual basis of wing C contained only one nonslack variable λ_D; but in the second stage solution, the optimal basis included new lambdas associated with observations from frontier wings A, I, J and G.

Note also that some of the lambda values in Table 4.9 are negative which indicates the lack of full envelopment of inefficient wings B, C, K and L. These negative lambdas correspond to the empirical observations added which replaced surplus vectors thereby minimizing the number of s_{rk}^{+} or s_{ik}^{-} variables in the optimal basis.

Proper Facet Detected for Wing K

In the original Charnes, Cooper and Rhodes DEA solution, wing K was evaluated relative to observations from wings F, G and N in its reference facet; but at the conclusion of the second stage evaluation $s+m-1 = 3+4-1 = 6$ lambdas were in wing K's optimal basis. Thus, wing K

Table 4.9

Wings in the Nearest Frontier Facet (Classified By Mission) of an Inefficient Wing and the Associated Lambda Values

Inefficient Wings	Frontier Combat Wings				Frontier Training Wings				Frontier Wings With Combat And Training Missions	
	A	D	H	I	E	F	J	N	G	M
B (Combat Wing)	$\lambda_A^* = .206$	$\lambda_D^* = .592$					$\lambda_J^* = -.279$	$\lambda_N^* = .146$		
C (Combat Wing)	$\lambda_A^* = -.889$	$\lambda_D^* = .158$		$\lambda_I^* = 1.56$			$\lambda_J^* = -.089$		$\lambda_G^* = .191$	
K (Training Wing)		$\lambda_D^* = -.258$			$\lambda_E^* = -.451$	$\lambda_F^* = .769$	$\lambda_J^* = -.939$	$\lambda_N^* = .446$	$\lambda_G^* = .932$	
L (Combat and Training Wing)		$\lambda_D^* = .058$	$\lambda_H^* = -.229$			$\lambda_F^* = -.153$		$\lambda_N^* = -.299$	$\lambda_G^* = .873$	

[1] Wing K has a proper facet determined by s+m-1 = 3+4-1 = 6 wings

was compared to a proper facet in this second stage analysis and all $\mu_{rk}^{(N)*}$ and $\nu_{ik}^{(N)*}$ values were greater than ϵ (see Table 4.8).

Range between Upper and Lower Bounds of Efficiency Indicates Degree of Nonenvelopment

Wing K is of particular interest because as a result of adding wings D, E and J in forming wing K's proper facet the efficiency measure decreased from .87 to the lower bound amount of .28 as shown in Table 4.10. Recall that the range between the upper and lower bounds of efficiency corresponds to the degree of nonenvelopment. Thus, wing K is an "outlier" unit and should receive special attention in any follow-up analysis by management.

To compute wing K's value if efficient as suggested in Property 2 of Chapter III, the lower bound efficiency measure of .28 would be used to adjust the vector of output observations by 1/.28 = 3.57 as follows:

Combat-Practice Sorties	3.57 (0.0)	=	0.0
Training Sorties	3.57(4,640.0)	=	16,564.8
Mission Capable Aircraft Days	3.57(4,562.0)	=	16,286.34
Available Aircraft	22.0	=	22.0
Supply Support	103.7	=	103.7
Labor Hours (× 1000)	704.0	=	704.0
Available Equipment	26,750.0	=	26,750.0

Table 4.10

Upper and Lower Bounds of Efficiency

1. Inefficient Wing j $j \epsilon \{B, C, K, L\}$	2. Optimal Value of Multiplier $\mu_{1j}^{(N)*}$ Associated With Output y_{1j} (Combat-Practice Sorties)	3. Observed Values For Output y_{1j}	4. Optimal Value of Multipliers $\mu_{2j}^{(N)*}$ Associated With Output y_{2j} (Training Sorties)	5. Observed Values For Output y_{2j}	6. Optimal Value of Multipliers $\mu_{3j}^{(N)*}$ Associated With Output y_{3j} (Mission Capable Aircraft Days)	7. Observed Values of y_{3j}	8. Minimum Efficiency $\omega_j^* = \sum_{r=1}^{3} \mu_{rj}^{(N)*} y_{rj}$ (Lower Bound)	9. CCR* DEA Efficiency h_j^* (Upper Bound)
B	.0000319	10,435	.0000318	0	.0000381	10,083	.72	.95
C	.0000558	13,991	.0000575	0	.0000020	14,552	.81	.87
K	.0000146	0	.0000191	4,640	.0000423	4,562	.28	.87
L	.0000000(є)	2,511	.0000047	5,021	.0000528	10,817	.60	.70

* Charnes, Cooper and Rhodes

The above adjusted output values imply the following rates if efficient:

$$\text{Sortie rate if efficient} = \frac{16{,}564.8}{12 \times 22} = 62.75$$

$$\text{Mission capable rate if efficient} = \frac{16{,}286.3 \times 100}{365 \times 22} = 202.8\%$$

Clearly, these rates if efficient are unattainable. It is highly unlikely that aircraft which have been flying at a sortie rate of 17.5 could sustain a sortie rate of 62.75, and it is impossible to achieve a mission capable rate greater than 100 percent. Thus, the .28 lower bound efficiency measure is inappropriate for computing values if efficient.

However, the comparison of wing K with frontier units shown in Table 4.11 suggests that the .87 upper bound efficiency value provided by the Charnes, Cooper and Rhodes DEA trial is likewise inappropriate. The data in Table 4.11 were obtained by dividing all input and ouput observed values of each wing in the table by that wing's observed value of daily average available aircraft, which in effect scales the wing observations to facilitate comparison. Such scaling, which is equivalent to multiplying primal constraints of the DEA and second stage models by

Table 4.11

Wing K Observed Values Per Aircraft Compared With the Observed and Average Values Per Aircraft of Efficient Wings

	1 Wings	2 Average Sorties Flown Per Aircraft (Annually)*	3 Mission Capable Aircraft Days Per Aircraft (Annually)	4 Daily Average Available Aircraft Per Aircraft (Annually)	5 Supply Support Factor Per Aircraft (Annually)	6 Available Labor Hours (x 1000)Per Aircraft (Annually)	7 Mission Essential Equipment Availability Per Aircraft (Annually)
Efficient Wings Defining Wing K's Facet	D	242.12	270.02	1.00	.25	29.33	1085.78
	E	204.68	257.94	1.00	.21	29.86	1190.48
	F	187.33	246.06	1.00	.20	25.38	1105.77
	G	196.55	263.25	1.00	.40	20.34	1171.88
	J	200.62	227.52	1.00	.08	26.94	1125.00
	N	296.94	265.71	1.00	.31	39.29	1125.00
Other Efficient Wings On the Frontier	A	211.00	219.36	1.00	.08	27.50	1125.00
	H	202.21	308.42	1.00	.79	28.00	1125.00
	I	222.36	224.94	1.00	.18	27.50	1108.33
	M	110.37	162.65	1.00	.09	14.74	1125.00
Averages For Wings Defining Facet		221.37	255.08	1.00	.24	28.52	1133.99
Averages For All Efficient Wings		207.42	244.59	1.00	.26	26.89	1128.72
Wing K's Observed Values (Per Aircraft)		210.91	207.36	1.00	4.71	33.64	1215.91

*Average Sorties Flown Per Aircraft During the Year = $\left[\left(\text{Annual Combat-Practice Sorties Flown}\right) + \left(\text{Annual Training Sorties Flown}\right)\right] \div \left(\text{Average Daily Aircraft Available During The Year}\right)$

a constant, does not change the optimal values of the u, v, μ, or ν multipliers presented earlier in this chapter. The data relationships in Table 4.11 indicate that wing K is indeed an outlier, and that the upper bound of .87 is an overestimation of its efficiency.

The wing K amounts in columns 3, 5, 6 and 7 are outliers in the sense that they are extreme or nearly extreme when compared to the ranges of values for frontier units. The wing K value for the average number of mission capable aircraft days per aircraft (column 3) is lower than all the other values associated with efficient wings in column 3. The supply support factor per aircraft for wing K is substantially higher than any of the other values in column 5. Similarly, the amount of labor hours per aircraft available to wing K during the year was the second highest amount in column 6; and wing K's availability of mission essential equipment per aircraft was the highest amount in column 7.

In short, wing K performed poorly in achieving a mission capable rate that was too low relative to frontier units while its input amounts for equipment, labor and supplies were too high. Furthermore, the mix of inputs at wing K is quite different from other wings because

of its extremely high outlier value for supply support. The Charnes, Cooper and Rhodes DEA solution assigned ϵ values to the multipliers associated with supply support and equipment availability (see Table 4.7, column 6) which in effect ignores these relatively high outlier values in order to achieve the .87 upper bound efficiency estimate. The true efficiency of wing K is lower than .87, but one is unable to determine how much lower at this point in the analysis; but, as stated earlier, the .28 lower bound measure is too low. Thus, it is impossible to determine the degree of inefficiency in wing K by either method.

Perhaps after closer inspection of wing K, knowledgeable managers could subjectively estimate the degree of wing K's inefficiency, which might enable the development of a "phantom" frontier unit for inclusion in the neighborhood or facet of wing K. This artificial unit could be given the same mix of inputs as wing K or a different mix if equipment and supplies need to be transferred. The inclusion of this artificial, phantom frontier unit in the facet of wing K should be constructed so that it produces an efficiency measure for wing K which is equal to the subjective estimate provided by managers.

In summary, a large difference between the upper and lower bounds of efficiency for any given wing implies

that a closer inspection of this outlier wing is needed before conclusions can be drawn about its actual degree of inefficiency.

On the other hand, for some not-fully-enveloped inefficient wings like C and L, the difference between the upper and lower bound measures is relatively small (see Table 4.10), i.e., these wings come closer to achieving full envelopment than did wing K. In such cases of near envelopment, the upper and lower bound measures provide better estimates of the actual degrees of inefficiency.

Marginal Rates of Substitution and Productivity

Regardless of the degree of nonenvelopment, the marginal rates of substitution and marginal rates of productivity obtained from the second stage process provide information about the frontier that is valuable even when analyzing outlier units like K. These rates are useful and informative because they are derived from the nearest set of *empirical* observations. Table 4.12 presents marginal rates of substitution and productivity for the facet associated with wing K.

Table 4.12

Marginal Rates of Substitution and Marginal Rates of Productivity

in the Proper Facet of Wing K

	1 ∂y_1 ($\mu_{1k}^{(3)*}$ = .0000146) (Combat-Practice Sorties)	2. ∂y_2 ($\mu_{2k}^{(3)*}$ = .0000191) (Flight Training Sorties	3. ∂y_3 ($\mu_{3k}^{(3)*}$ = .0000423) (Mission Capable Aircraft Days)	4. ∂x_1 ($\nu_{1k}^{(3)*}$ = .0074578) (Average Available Aircraft)	5. ∂x_2 ($\nu_{2k}^{(3)*}$ = .0067169) (Supply Support) Factor)	6. ∂x_3 ($\nu_{3k}^{(3)*}$ = .0001566) (Available Labor Hours)	7. ∂x_4 ($\nu_{4k}^{(3)*}$ = .000011) (Mission Essential Equipment Availability
1. $\partial y_1(\mu_{1k}^{(3)*}$ = .0000146)		- 0.76398	- 0.34515	0.00196	0.00217	0.09323	13.27273
2. $\partial y_2(\mu_{2k}^{(3)*}$ = .0000191)	- 1.30822*		- 0.45154	0.00256	0.00284	0.12197	17.36364
3. $\partial y_3(\mu_{3k}^{(3)*}$ = .0000423)	- 2.89726	- 2.21466		0.00567	0.00630	0.27011	38.45455
4. $\partial x_1(\nu_{1k}^{(3)*}$ = .0074578)	510.80822	390.46073	176.30733		- 1.11030	- 47.62324	-6,779.81818
5. $\partial x_2(\nu_{2k}^{(3)*}$ = .0067169)	460.06164	351.67016	158.79196	- 0.90065		- 42.89208	-6,106.27273
6. $\partial x_3(\nu_{3k}^{(3)*}$ = .0001566)	10.72603	8.19895	3.70213	- 0.02100	- 0.02331		- 142.36364
7. $\partial x_4(\nu_{4k}^{(3)*}$ = .0000011)	0.07534	0.05759	0.02600	- 0.00015	- 0.00016	- 0.00702	

* Note: The number shown in column i and row j is the partial derivative of the column variable with respect to the row variable; e.g., the number in column 1 row 2 is:

$$\frac{\partial y_1}{\partial y_2} = - \frac{\mu_{2k}^{(3)*}}{\mu_{1k}^{(3)*}} = - \frac{.0000191}{.0000146} = - 1.3082.$$

The negative values in Table 4.12 are the marginal rates of substitution and the positive values are the marginal rates of productivity. For example, the value -1.30822 in column 1, row 2 of Table 4.12 indicates the marginal rate of substitution between combat-practice sorties (y_1) and training sorties (y_2). Thus, if unit K is operating efficiently, an increase of ten training sorties would require a decrease of approximately 10 × (1.30822) ≈ 13 combat-practice sorties, provided all other input and output amounts remain constant.

For wings B, C and K, the marginal rates of substitution of combat-practice sorties with respect to training sorties are all nearly equal to one in their respective facets; i.e., using the multiplier data from Table 4.8:

Wing B Facet (4.8)

$$\left(\frac{\partial y_1}{\partial y_2}\right)_B = - \frac{\mu_{2B}^{(N)*}}{\mu_{1B}^{(N)*}} = - \frac{.0000318}{.0000319} = - .997$$

Wing C Facet

$$\left(\frac{\partial y_1}{\partial y_2}\right)_C = - \frac{\mu_{2C}^{(N)*}}{\mu_{1C}^{(N)*}} = - \frac{.0000575}{.0000558} = - 1.03$$

Wing K Facet

$$\left(\frac{\partial y_1}{\partial y_2}\right)_K = -\frac{\mu_{2K}^{(N)*}}{\mu_{1K}^{(N)*}} = -\frac{.0000191}{.0000146} = -1.31$$

Thus, combat-practice sorties and training sorties trade off nearly one for one in each of the facets. The trade off appears to be realistic since the sortie values used in this example were actual amounts flown by real tactical fighter wings. The amounts used were obtained from an FY81 Tactical Air Command report [44].

The marginal rates of substitution obtained via the Charnes, Cooper and Rhodes DEA model are not realistic. In Table 4.7, wings B and K were the only inefficient wings having nonepsilon multipliers for both combat-practice sorties and training sorties. Using these multiplier values, the following marginal rates of substitution were obtained:

Wing B Facet

$$\left(\frac{\partial y_1}{\partial y_2}\right)_B = -\frac{.000074}{.000090} = \quad .822$$

Wing K Facet

$$\left(\frac{\partial y_1}{\partial y_2}\right)_K = -\frac{.000173}{.000009} = -19.22$$

The above rates are significantly different from those resulting from the second stage evaluation. The wing K rate of -19.22 seems particularly excessive. If all other input and output values remain constant and if a wing is operating at peak efficiency, then one would expect that an increase of ten training sorties would require a reduction of about ten combat-practice sorties since training sorties and combat-practice sorties require nearly the same amount of resources.

Furthermore, the data in row 1, columns 1, 2 and 3 of Table 4.12 provide the marginal values of one additional aircraft ($\Delta x_1 = 1$) in increasing each of the outputs; e.g., if wing K gains one additional aircraft, then to remain on the frontier the wing should produce about 511 additional combat-practice sorties during the next year, provided of course that all other inputs and outputs remain unchanged. The remaining data in Table 4.12 could be used in similar fashion to evaluate the impact of other changes in input or output amounts.

In the next chapter, a network graph will be presented which illustrates the use of optimal second stage multipliers in preserving frontier marginal rates of substitution and productivity in an allocation problem.

CHAPTER V

CONCLUSIONS OF THE STUDY AND DIRECTIONS FOR FURTHER RESEARCH

Introduction

The public expects military efficiency from the combat forces it supports with tax dollars. For several years, the United States Air Force and the other military services have been searching for integrative models of efficiency and capability, models which will aid in the detection and diagnosis of operational problems as well as assist in budgeting and other forms of planning.

Many of the efficiency related modeling forms reviewed or adopted by the Air Force have been either simulation models designed to project expected levels of military capability given specific resource mixes or mathematical programming approaches to predict the frontiers of productive potential of these mixes. The latter is particularly difficult since the data from which such projections must be made is historical in nature and contains observations related to inefficient as well as efficient processes. Observations from efficient operations must

be detected first to provide a basis for predictions of productive potential.

Furthermore, too little is known of the underlying productive processes in military operations and, therefore, it is difficult to specify mathematical relationships which represent these processes. As a result, the Air Force must rely on relative measures of performance derived from empirical data, and preferably these measures of performance should be derived without making a priori assumptions about the mathematical forms of the underlying production functions.

Summary of Study Results

Many of the analytical techniques currently used by the Air Force, such as ratio analysis, do not require such a priori specification of functional forms, but these techniques are equally unattractive for other important reasons. For example, ratio analysis requires the use of partial measures of performance which are unable to take into account interactions and trade offs over the full range of inputs and outputs of a process making it difficult to compare the performance of units using the process. Such partial measures can cause units to be incorrectly

classified as efficient or inefficient by focusing on one or a few of many important factors and overlooking others which are relevant in establishing neighborhoods of comparison; i.e., the use of partial measures can lead to incorrect assessments of performance as a result of inadvertent omission of relevant observations.

Regression is another analytical technique which has been commonly used by the Air Force for estimating relationships between the inputs and outputs of a production process. Regression does require an assumption about the mathematical form of the production function (e.g., linear, multinomial, log-linear, etc.), and in many applications is used to provide estimates of average relationships which are uninformative for frontier estimation purposes.

At present, regression techniques are largely inappropriate for estimating the frontiers of efficiency and productivity of public service agencies. However, given that frontier units can be detected by some other method, regression might be useful in follow-on investigations to predict relationships based on efficient observations only, provided of course the sample of efficient observations is large enough to make the regression results meaningful. By using only the efficient observations,

regression equations are obtained which come closer to accurately representing frontiers and which have no non-random inefficient behavior subsumed in the residual term.

Because of the aforementioned limitations of ratio analysis and regression, the efficiency measurement concepts of Farrell [27] and the subsequent formulation of the Charnes, Cooper and Rhodes Data Envelopment Analysis (DEA) model [19] provided a much better approach for evaluating the efficiency of multiinput, multioutput public service organizations, and provided a basis for further development into other areas of analysis and management planning. In fact, the Charnes, Cooper and Rhodes DEA model served as the basic starting point for this study. Their model enables the unified analysis of multiple technical, economic and effectiveness measures in contrast to past reliance by the Air Force on "partial measures" of productivity. This DEA model makes no assumptions about industry-wide production functions, but uses empirical observations to measure efficiency relative to local frontiers. No claim needs to be made for demonstrated causality between inputs and outputs since unspecified processes are the causal agent and the model allows for an unknown amount of inefficiency to exist.

Furthermore, the Charnes, Cooper and Rhodes method takes all outputs and inputs into account simultaneously including differences in input/output mixes and trade offs among factors. It indicates which organizations are on the efficiency frontier, establishes a piece-wise linear approximation of the frontier surface using efficient units, and assigns an efficiency measure based on how far the unit is from a frontier point directly between the unit and the origin, a point for which input and output values are linear combinations of the observations from an efficient set of "neighborhood" organizations both real and artificial. Limited evaluation of frontier points, neighborhoods, and upper bound efficiency measures for individual units are accessible through the Charnes, Cooper and Rhodes DEA method.

Unfortunately, their method is not a remedy for all efficiency analysis difficulties. It can indicate which units are efficient and which are not; and, if the efficiency measure of a unit is obtained from a full neighborhood set of s+m-1 observations (i.e., fully enveloped), where s is the number of outputs and m is the number of inputs, then the DEA rating is an accurate representation of the degree of inefficiency in the unit. But in all

reported demonstrations or implementations in multiple input and multiple output situations known to the author, the condition of being fully enveloped never occurs.

The lack of full envelopment is indicated by the Charnes, Cooper and Rhodes model in the form of positive amounts of slack or surplus at optimality, and sometimes these amounts are large causing significant overestimations of efficiency and misleading information about how to achieve the frontier. The positive slack or surplus amounts and the associated overestimations of efficiency are accompanied by frontier comparison sets (neighborhood facets) with too few elements and by rates of substitution and production among inputs and outputs in these facets which are not derived entirely from empirical observations, conditions which often lead to erroneous conclusions.

This study was undertaken to identify the closest complete set of empirical frontier observations, or a maximum number of observations if a complete set is not achievable. These frontier observations serve as a basis for determining approximate rates of substitution and productivity in the neighborhood of an inefficient unit and provide a range of efficiency for the unit. Such

empirically based information about the nearest frontier region or facet is needed so that additional models can be formulated to explore alternative mixes or levels of inputs or outputs in this region.

In Chapter III, a second stage model was presented for locating as many frontier units as possible defining the nearest facet thus enabling the computation of a lower bound of efficiency for not-fully-enveloped units. The second stage model also provides information about rates of substitution and marginal productivity among inputs and outputs. These extensions were tested in Chapter IV on a three output, four input Air Force wing evaluation problem, and the results obtained were consistent with the theoretical expectations.

The second stage technique identified more neighborhood frontier units than the Charnes, Cooper and Rhodes DEA model for every inefficient unit in the Air Force example; and, as expected, the efficiency measures for these units decreased as the frontier facets were extended.

Potential for Further Research into Resource Allocation and Goal Setting

The developments in this study provide a few additional analytical capabilities suitable for use by the Air Force in the analysis and interpretation of efficiency and the preparation of management plans. As stated above, these extensions include methods for post DEA efficiency analysis to detect rates of substitution and marginal productivities in nearby frontier facets, facets which are formed from as many empirically observed values as possible. Such methods are needed in developing resource allocation models and in establishing realistic output expectations in management plans. The connection between the above methods and the problem of resource allocation will now be illustrated through the use of network representations of the key mathematical relationships in frontier facets.

Resource Allocation

Recall that at the final iteration (N) of the second stage evaluation all frontier units in $E_K^{(N)}$ associated with the facet of an inefficient unit k satisfy the following equality of primal model (3.22):

$$\sum_{r=1}^{s} \mu_{rk}^{(N)*} y_{rj} - \sum_{i=1}^{m} \nu_{ik}^{(N)*} x_{ij} = 0, \text{ for every } j \varepsilon E_k^{(N)} \quad (5.1)$$

To simplify notation, let $\mu_{rk}^{(N)*} \equiv \mu_{rk}^{*}$ and $\nu_{ik}^{(N)*} \equiv \nu_{ik}^{*}$. Using the generalized network graphing conventions of Glover, Hultz, Klingman and Stutz [30], the mathematical relationship at (5.1) can be represented by the network graph in Figure 5.1.

This figure is one component of a larger reallocation network, where the total network includes all of the subordinate firms (decision making units) belonging to a conglomerate which is subject to the allocation decisions of a single headquarters. The physical inputs of each firm or subunit could then be thought of as resources, some of which could be redistributed among the firms in an attempt to improve the overall productivity of the conglomerate. Thus, physical inputs (e.g., people, supplies and equipment) might flow from one firm to another as a result of the allocation decisions of top level managers.

Arc paths in the network represent the possible transfers of commodities or assets. These transfers would also have unit costs associated with them, and management would probably specify upper and lower bounds on the amounts that could be transferred and similar limits on

Graphing Conventions [30]:

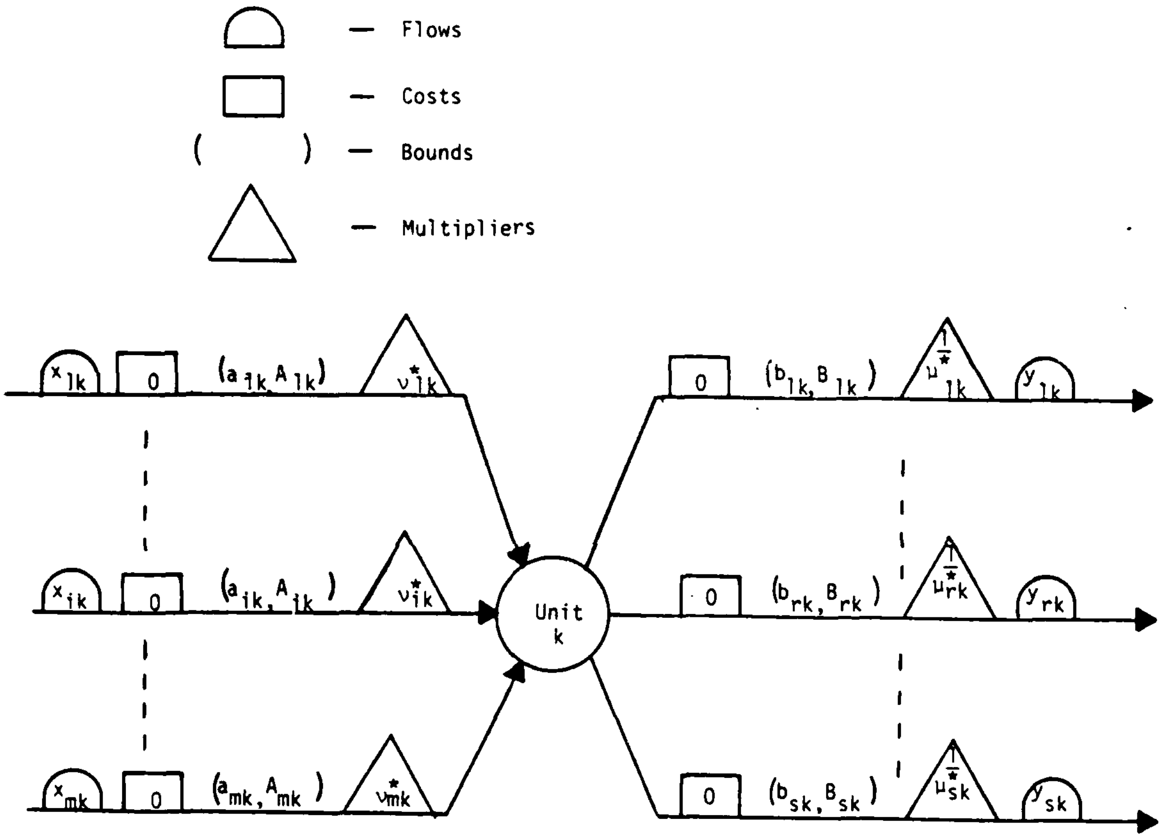


Figure 5.1

<u>Network Graph of Frontier Facet for Unit k</u>

the amounts required to be on-hand at specific locations. Furthermore, the flows traversing the arcs could be acted on by multipliers which transform the flows into other units of measure; e.g., "number of laborers" could be transformed into "labor hours."

Figure 5.1 illustrates how the concepts of arc paths, flows, costs and multipliers can be used to model the equality relation at (5.1). This figure also demonstrates how necessary parameters are supplied by the second stage evaluation.

The x_{ik} flows in Figure 5.1 represent allocations of resources to unit k from some earlier portion of the overall network and the y_{rk} flows are the outputs resulting from efficient use of the allocated amounts x_{ik}. The costs are assumed to be zero implying that any costs associated with transfer of resources from one unit to another have somehow been considered in earlier portions of the overall model.

The bounds limiting the input and output amounts on the arcs are determined from the range of each input and output value in the frontier facet of unit k as follows:

Output Bounds:

$$b_{rk} = \min_{j \in E_k^{(N)}} \{y_{rj}\}, \quad B_{rk} = \max_{j \in E_k^{(N)}} \{y_{rj}\}$$

$r = 1,2,\ldots,s$

Input Bounds:

$$a_{ik} = \min_{j \in E_k^{(N)}} \{x_{ij}\}, \quad A_{ik} = \max_{j \in E_k^{(N)}} \{x_{ij}\}$$

$i = 1,2,\ldots,m$

Thus, the allocation of resources x_{ik} and the output values produced y_{rk} are restricted to the following ranges determined from the frontier facet:

$$a_{ik} \leq x_{ik} \leq A_{ik}, \quad i = 1,2,\ldots,m$$

$$b_{rk} \leq y_{rk} \leq B_{rk}, \quad r = 1,2,\ldots s$$

Of course, other bounds could be substituted if these values were not considered to be feasible for unit k.

The multipliers on the arcs, ν_{ik}^* and $1/\mu_{rk}^*$, preserve the relationship at (5.1) above as follows. The total flow into the unit k node is:

$$\sum_{i=1}^{m} \nu_{ik}^* x_{ik}$$

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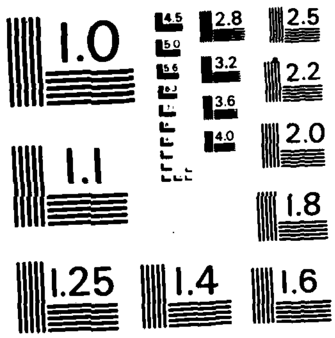

MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

The flow on a single output arc r out of the unit k node must be $\mu^*_{rk}y_{rk}$ in order that the flow

$$y_{rk} = \frac{1}{\mu^*_{rk}}(\mu^*_{rk}y_{rk})$$

will occur after the multiplier. Thus the total flow leaving the unit k node is:

$$\sum_{r=1}^{s} \mu^*_{rk}y_{rk}.$$

Finally, flow conservation at the unit k node preserves the mathematical relation at (5.1) above.

The network graph in Figure 5.1 should be thought of as one component in a larger resource allocation network which guarantees that the vector of resource allocation to unit k $(x_{1k},x_{2k},\ldots,x_{mk})$ and the vector of planned outputs $(y_{1k},y_{2k},\ldots,y_{sk})$ form an efficient combination within the neighborhood facets formed by those frontier units.

Goal Setting

Efficiency is not the only criterion of interest in an allocation problem for public service agencies. Effectiveness goals must also be addressed. Perhaps

several agencies, including some of the frontier units, have been producing levels of output which are considered to be too low by managers. Perhaps goals should be set in an attempt to increase the levels of achievement of units. Figure 5.2 shows one possible approach in establishing an output goal for all of the $j = 1,2,\ldots,n$ units in the industry. This figure represents an objective of having the average of output 1 of the $j = 1,2,\ldots,n$ units equal or exceed the particular average output amount G_1 desired by management. The values y_{ij}, $j = 1,2,\ldots,n$ are the amounts of output 1 of each unit j. The curved arc having flow s^+ is a surplus arc and the arc s^- is a slack arc. One or the other of these flows will be nonzero whenever the average of the output amounts

$$\frac{\sum_{j=1}^{n} y_{ij}}{n}$$

arriving at the output 1 node is not equal to G_1. If the average is greater than G_1, then the surplus flow s^+ will be greater than zero with no penalty (a cost of zero). If the average is less than G_1, then s^- will be greater than zero with a high penalty. Thus the model would

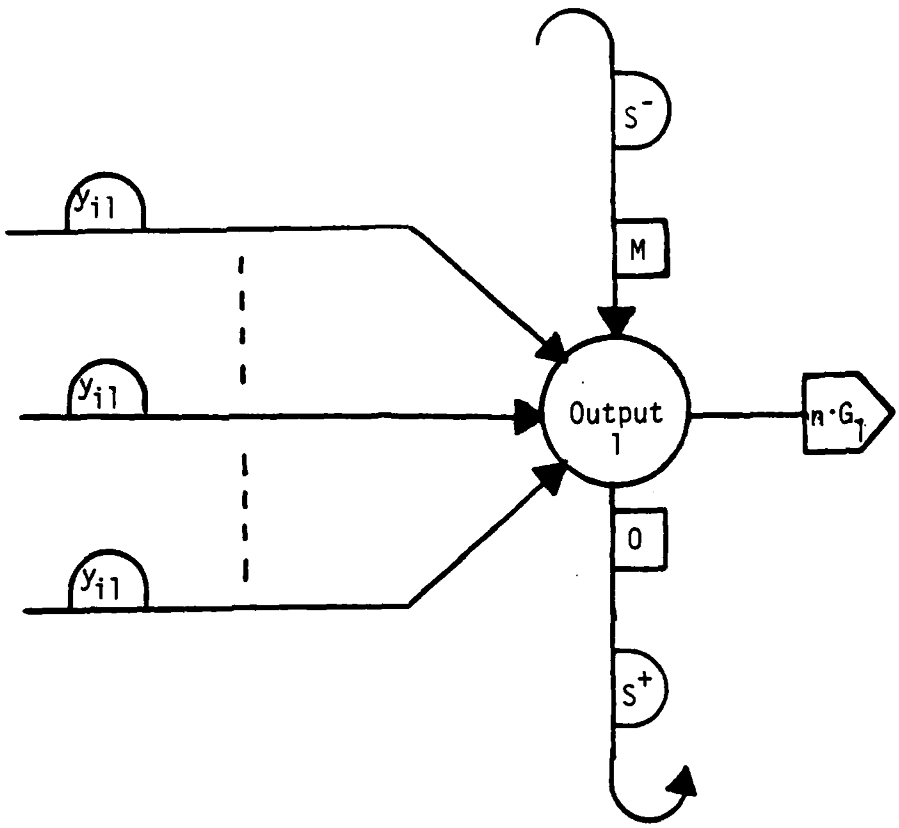

Figure 5.2

Goal of Achieving an Average Value
For Output 1 Greater Than
a Desirable Amount G_1

prefer to achieve an average amount of output 1 greater than the G_1 goal desired by management.

The author has been unable to pursue the allocation and goal setting problems much further than the limited points presented here. But further study in these areas would prove to be very beneficial and enlightening. A complete network representation of the allocation problem is needed with a collection of goal setting formulations (perhaps nonnetwork) which address a variety of possible goals affecting output levels, or output mix or both.

Concluding Remarks

The Air Force unquestionably needs models to assist in the identification of inefficiencies and to enable the development of plans to bring about technological changes or resource reallocations which improve the collective efficiency of combat units and which help guarantee the achievement of acceptable levels of military preparedness. The Charnes, Cooper and Rhodes Data Envelopment Analysis model plus the second stage extensions presented in this study and the suggested further developments into areas of resource allocation and goal setting are all

worthwhile directions for research, the outcomes of which would be of considerable value to the Air Force and other public service agencies. It is hoped that these research directions will lead to field implementations of management systems which promote greater efficiency throughout the public service domain and in so doing help feed more starving children, hire more willing workers, educate more deserving students while continuing to maintain our nation's military strength and security.

APPENDIX 1

DEA MODEL AND SOLUTION FOR DECISION-MAKING UNIT 61

(See Test Case, Chapter III)

```
MPOS VERSION 2.0       NORTHWESTERN UNIVERSITY

     ************************************
     *                                  *
     *              M P O S             *
     *                                  *
     *            VERSION 2.0           *
     *                                  *
     * MULTI-PURPOSE OPTIMIZATION SYSTEM *
     *                                  *
     ************************************

***** PROBLEM NUMBER  1 *****

       TITLE
       FIND BASIS FOR UNIT 61
       REGULAR
       VARIABLES
       U1 TO U3 V1 TO V10
       MAXIMIZE
       71.4U1+78.13U2+70U3
       CONSTRAINTS
   1.  90U1+86U2+87U3-45V1-65V2-30V3-3.51V4-4.06V5-99.24V6-100V7-
       89.85V8-86.49V9-83.78V10 .LE.0
   2.  90U1+90.25U2+88.8U3-45V1-65V2-30V3-3.15V4-3.74V5-90.67V6-100V7-
       88.15V8-87.8V9-91.46V10 .LE.0
   3.  84.2U1+86.5U2+85.6U3-45V1-55V2-30V3-3.83V4-4.02V5-95.9V6-100V7-
       73.88V8-78.08V9-82.19V10 .LE.0
   4.  68.6U1+69.63U2+70U3-45V1-65V2-30V3-3.09V4-5.67V5-94.86V6-100V7-
       65.99V8-28.30V9-47.17V10 .LE.0
   5.  77U1+78.13U2+75.2U3-45V1-65V2-30V3-3.44V4-4.37V5-96.23V6-80V7-
       71.6V8-58.93V9-74.11V10  .LE.0
   6.  82.7U1+84.63U2+84.6U3-45V1-65V2-30V3-3.16V4-4.65V5-94.63V6-100V7-
       74.9V8-93.52V9-96.3V10 .LE.0
   7.  87U1+85.25U2+85.8U3-45V1-65V2-30V3-3.03V4-3.95V5-95.57V6-50V7-
       79.75V8-84.54V9-90.72V10 .LE.0
   8.  80.5U1+81.25U2+78.2U3-45V1-65V2-30V3-3.45V4-3.75V5-96.23V6-60V7-
       79.39V8-53.49V9-64.34V10 .LE.0
   9.  76.3U1+78.13U2+77.6U3-45V1-85V2-25V3-6.26V4-3.43V5-97.14V6-100V7-
       78.56V8-49.12V9-56.14V10 .LE.0
  10.  77.6U1+83.38U2+74.8U3-60V1-90V2-30V3-5.47V4-3.61V5-96.76V6-66.67V7-
       80.74V8-42.5V9-68.75V10 .LE.0
  11.  75.7U1+72U2+74U3-60V1-90V2-60V3-6.33V4-3.34V5-98.21V6-100V7-
       87.9V8-37.61V9-50.43V10 .LE.0
  12.  86.1U1+92.13U2+91.4U3-35V1-90V2-20V3-21.49V4-4.01V5-96.92V6-100V7-
       78.63V8-90.97V9-84.03V10 .LE.0
  13.  79.7U1 +92.38U2+93U3-45V1-90V2-15V3-28.32V4-4.71V5-97.8V6-100V7-
       79.5V8-90.79V9-85.53V10 .LE.0
  14.  85.4U1+90.88U2+91.2U3-45V1-60V2-30V3-27.21V4-4.9V5-96.79V6-61.54V7-
       81.14V8-92.65V9-98.53V10 .LE.0
  15.  81.7U1+78.88U2+75.4U3-30V1-60V2-15V3-28.34V4-4.68V5-98.24V6-92.31V7-
       56.82V8-86.36V9-83.33V10 .LE.0
  16.  87.4U1+86.25U2+88.4U3-45V1-60.V2-30V3-31.63V4-4.73V5-97.89V6-61.54V7-
       76.72V8-84.85V9-74.24V10 .LE.0
  17.  79.8U1+85U2+85.2U3-45V1-60V2-30V3-34.29V4-4.69V5-96.5V6-47.06V7-
       84.55V8-84.71V9-94.12V10 .LE.0
  18.  73U1+66.75U2+67U3-60V1-90V2-15V3-23.62V4-6.97V5-98.86V6-100V7-
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```

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    57.53V8-75.71V9-74.29V10 .LE.0
20. 87U1+90.38U2+92U3-50V1-50V2-20V3-28.79V4-4.84V5-90.24V6-100V7-
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21. 84.8U1+88U2+89.8U3-40V1-65V2-15V3-34.8V4-3.95V5-96.57V6-100V7-
    91.1V8-99V9-97V10.LE.0
22. 74.3U1+79.25U2+81.4U3-60V1-120V2-30V3-35.5V4-3.83V5-94.1V6-50V7-
    72.85V8-11.54V9-85.26V10 .LE.0
23. 86U1+89.5U2+88U3-60V1-120V2-30V3-30.78V4-2.79V5-92.9V6-50V7-
    76.76V8-42.18V9-88.15V10 .LE.0
24. 85.1U1+82.38U2+81.8U3-60V1-120V2-30V3-19.84V4-4.42V5-97.59V6-28.57V7-
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25. 80.7U1+80.75U2+78.2U3-60V1-120V2-30V3-24.52V4-5.34V5-97.65V6-33.33V7-
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26. 78U1+84.63U2+83.2U3-60V1-120V2-30V3-41.45V4-3.77V5-91.43V6-100V7-
    73.61V8-28.4V9-80.25V10 .LE.0
27. 79.2U1+80.75U2+80.4U3-60V1-120V2-30V3-24.42V4-3.46V5-95.52V6-33.33V7-
    79.81V8-31.71V9-75V10 .LE.0
28. 73.9U1+77.13U2+71.8U3-60V1-90V2-30V3-23.16V4-4.33V5-96.1V6-100V7-
    78.35V8-15.49V9-22.54V10 .LE.0
29. 61.6U1+62.63U2+65.4U3-40V1-75V2-35V3-21.25V4-3.23V5-88.1V6-100V7-
    65.5V8-6.49V9-22.08V10 .LE.0
30. 82.3U1+90U2+90.4U3-40V1-60V2-30V3-12.76V4-4.41V5-95.54V6-100V7-
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31. 78.6U1+86U2+84.6U3-60V1-75V2-20V3-7.05V4-4.02V5-96.99V6-100V7-
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33. 86.7U1+90.88U2+90.6U3-40V1-60V2-60V3-12.48V4-4.14V5-95.8V6-94.34V7-
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43. 70.6U1+58.36U2+56.6U3-50V1-55V2-25V3-22.08V4-4.71V5-94.57V6-66.67V7-
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45. 81.8U1+83.88U2+79.6U3-45V1-90V2-20V3-62.65V4-3.57V5-96.57V6-50V7-
    74.15V8-78.95V9-84.21V10 .LE.0
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```

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54.   90.3U1+87.88U2+91.4U3-69V1-30V2-90V3-87.77V4-7.16V5-97.38V6-100V7-
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56.   90.5U1+96.63U2+93.6U3-40V1-60V2-25V3-19.38V4-4.86V5-95.57V6-100V7-
      93.93V8-97.37V9-100V10 .LE.0
57.   98.2U1+95.15U2+92.8U3-50V1-70V2-20V3-12.68V4-4.1V5-97.29V6-100V7-
      92.83V8-91.84V9-100V10 .LE.0
58.   89U1+95.25U2+93.2U3-55V1-65V2-20V3-15.47V4-4.02V5-91.77V6-100V7-
      91.96V8-95.12V9-100V10 .LE.0
59.   92.3U1+91.13U2+91.4U3-40V1-75V2-20V3-12.67V4-3.7V5-99.7V6-75V7-
      96.876V8-96.12V9-100V10 .LE.0
60.   77.1U1+80.75U2+80U3-45V1-80V2-15V3-12.46V4-4.02V5-96.5V6-75V7-
      26.63V8-70.79V9-100V10 .LE.0
61.   84.8U1+90.75U2+91.4U3-45V1-75V2-25V3-13.87V4-4.23V5-99.29V6-75V7-
      66.2V8-79.83V9-100V10 .LE.0
62.   84.1U1+90.38U2+84.6U3-40V1-70V2-25V3-13.2V4-4.48V5-97.07V6-75V7-
      77.02V8-75V9-100V10 .LE.0
63.   94.3U1+96.75U2+95.2U3-45V1-65V2-20V3-24.97V4-3.75V5-97.71V6-100V7-
      86.49V8-95.31V9-100V10 .LE.0
64.   88.5U1+95.75U2+95U3-45V1-70V2-30V3-12.86V4-4.27V5-87.36V6-100V7-
      93.6V8-89.01V9-100V10 .LE.0
65.   78.3U1+86.5U2+84.0U3-50V1-60V2-25V3-12.91V4-3.38V5-98.86V6-100V7-
      67.07V8-87.5V9-100V10 .LE.0
66.   91.5U1+95.38U2+93.2U3-40V1-65V2-30V3-17.95V4-3.79V5-97.43V6-100V7-
      93.36V8-100V9-100V10 .LE.0
67.   84.4U1+94.25U2+90.4 U3-50V1-70V2-25V3-17.68V4-3.72V5-97V6-50V7-
      87.91V8-90.2V9-100V10 .LE.0
68.   91.1U1+92U2+87.2U3-55V1-65V2-20V3-15.27V4-3.5V5-84.29V6-100V7-
      79.02V8-96.43V9-100V10 .LE.0
69.   91.6U1+96.63U2+97U3-40V1-70V2-25V3-14.82V4-3.43V5-98.29V6-100V7-
      91.41V8-96V9-100V10 .LE.0
70.   89.4U1+93.0U2+90.6U3-45V1-70V2-20V3-16.7V4-4.11V5-98.19V6-66.67V7-
      84.93V8-97.14V9-100V10 .LE.0
71.   87.2U1+93.5U2+90.6U3-50V1-65V2-25V3-15.99V4-4.08V5-97.14V6-100V7-
      67.83V8-84.04V9-100V10 .LE.0
72.   94.4U1+96U2+96.4U3-40V1-65V2-25V3-13.57V4-4.3V5-98.21V6-100V7-
      84.94V8-85.06V9-100V10 .LE.0
73.   91.8U1+95.75U2+94U3-40V1-70V2-20V3-14.97V4-4.2V5-97.89V6-100V7-
      93.7V8-85.59V9-100V10 .LE.0
74.   88.5U1+93.25U2+92U3-45V1-60V2-45V3-20.57V4-3.98V5-97.54V6-80V7-
      65.79V8-87.4V9-90.55V10 .LE.0
75.   75.5U1+83.5U2+79.8U3-55V1-55V2-15V3-28.28V4-4.86V5-97.66V6-93.46V7-
      69.93V8-60.71V9-66.67V10 .LE.0
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 76.  95.9U1+93.U2+91.8U3-45V1-45V2-25V3-27.09V4-3.85V5-98.57V6-94.89V7-
      87.87V8-93.02V9-95.35V10 .LE.0
 77.  73.4U1+82.6U2+80U3-40V1-40V2-20V3-26.9V4-4.07V5-98.13V6-81.74V7-
      72.28V8-73.26V9-79.07V10 .LE.0
 78.  92.2U1+87.13U2+86.6U3-60V1-120V2-20V3-37.31V4-5.08V5-97.79V6-85.84V7-
      49.86V8-72.73V9-97.73V10.LE.0
 79.  96.7U1+96.25U2+96.4U3-45V1-90V2-20V3-31.36V4-4.02V5-99.38V6-95.71V7-
      78.75V8-82.72V9-96.3V10.LE.0
 80.  91U1+81.38U2+75.6U3-40V1-60V2-30V3-38.33V4-5.45V5-96.08V6-68.65V7-
      22.12V8-67.11V9-81.58V10 .LE.0
 81.  92U1+93.75U2+90.8U3-60V1-90V2-15V3-40.17V4-4.52V5-99.51V6-100V7-
      82.55V8-95.83V9-97.92V10 .LE.0
 82.  63.7U1+74.38U2+71U3-60V1-100V2-20V3-42.25V4-4.09V5-93.39V6-48.12V7-
      36.38V8-44.22V9-65.99V10 .LE.0
 83.  95.9U1+94.75U2+95U3-45V1-100V2-20V3-37.03V4-4.64V5-97.54V6-100V7-
      85.19V8-88.06V9-95.52V10 .LE.0
 84.  87.9U1+93.38U2+94.2U3-50V1-90V2-20V3-34.76V4-3.04V5-91.16V6-92.38V7-
      72.62V8-91.76V9-96.47V10 .LF.0
 85.  94U1+90.63U2+91.4U3-45V1-90V2-15V3-33.33V4-4.14V5-95.42V6-98.72V7-
      85.4V8-94.74V9-100V10 .LE.0
 86.  89U1+88.25U2+87.8U3-45V1-100V2-20V3-42.33V4-4.45V5-97.61V6-40.91V7-
      50.23V8-85.09V9-96.49V10 .LE.0
 87.  86.3U1+90.63U2+88U3-45V1-90V2-20V3-36.26V4-4.58V5-94.61V6-85.71V7-
      59.43V8-80.56V9-88.89V10 .LE.0
 88.  71.9U1+64.75U2+61.8U3-60V1-75V2-30V3-36.85V4-4.31V5-97.4V6-33.33V7-
      2.3V8-47.17V9-55.66V10 .LE.0
 89.  80.9U1+85.63U2+82.8U3-40V1-60V2-15V3-35.56V4-4.3V5-95.79V6-72.73V7-
      51.53V8-73.98V9-85.37V10 .LE.0
 90.  83.9U1+83.25U2+83U3-45V1-75V2-20V3-30.4V4-4.34V5-96.49V6 -100V7-
      37.95V8-77.92V9-90.91V10 .LE.0
 91.  76.8U1+84.5U2+76.8U3-45V1-100V2-20V3-34.74V4-4.17V5-92.67V6-100V7-
      45.15V8-86.87V9-77.78V10 .LE.0
 92.  78.9U1+75.1U2+73.4U3-55V1-55V2-30V3-45.62V4-4.44V5-96.78V6-90.91V7-
      34.18V8-53.1V9-66.37V10 .LE.0
 93.  90U1+90.25U2+87.6U3-60V1-105V2-15V3-38.58V4-5.83V5-95.41V6-71.43V7-
      68.79V8-83.5V9-96.12V10 .LE.0
 94.  71.7U1+74.5U2+74.6U3-55V1-130V2-21V3-15.81V4-4.64V5-96.95V6-100V7-
      79.87V8-3.51V9-10.53V10 .LE.0
 95.  85.9U1+92.13U2+94.6U3-50V1-90V2-20V3-15.55V4-3.82V5-99.05V6-66.67V7-
      75.82V8-87.84V9-82.43V10 .LE.0
 96.  84U1+86.63U2+89U3-50V1-50V2-30V3-15.99V4-4.25V5-93.03V6-80V7-
      70.22V8-80.91V9-74.55V10 .LE.0
 97.  69.3U1+71.5U2+72.6U3-55V1-55V2-30V3-13.31V4-3.96V5-96.9V6-100V7-
      83.01V8-27.38V9-30.95V10 .LE.0
 98.  87.8U1+86.13U2+97.6U3-30V1-120V2-20V3-14.54V4-3.63V5-98.86V6-83.33V7-
      63.72V8-40V9-26.67V10 .LE.0
 99.  75.5U1+81.5U2+80U3-45V1-60V2-30V3-13.62V4-4.79V5-97.86V6-75V7-
      85.63V8-44.16V9-53.25V10 .LE.0
100.  72.3U1+74.5U2+71.6U3-50V1-90V2-20V3-17.85V4-4.68V5-96.38V6-100V7-
      79.71V8-1.64V9-22.95V10 .LE.0
101.  74.1U1+74.88U2+75.8U3-60V1-90V2-40V3-11.29V4-4.35V5-96.14V6-100V7-
      63.04V8-21.51V9-49.46V10 .LE.0
102.  64.4U1+67.13U2+68.4U3-60V1-60V2-45V3-22.51V4-3.49V5 -94.29V6-100V7-
      100V8-57.14V9-23.38V10 .LE.0
103.  74U1+78.5U2+74.6U3-60V1-60V2-45V3-25.6V4-3.51V5-95.86V6-100V7-
      98.25V8-66.67V9-47.62V10 .LE.0
104.  50V1+90V2+20V3+21.91V4+5.21V5+90.57V6+100V7+100V8+92.86V9+
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      85.71V10.EQ.1
105.  U1.GE..000001
106.  U2.GE..000001
107.  U3.GE..000001
108.  V1.GE..000001
109.  V2.GE..000001
110.  V3.GE..000001
111.  V4.GE..000001
112.  V5.GE..000001
113.  V6.GE..000001
114.  V7.GE..000001
115.  V8.GE..000001
116.  V9.GE..000001
117.  V10.GE..000001
      RNGRHS
      OPTIMIZE
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USING REGULAR
FIND BASIS FOR UNIT 61

SUMMARY OF RESULTS

VAR NO	VAR NAME	ROW NO	STATUS	ACTIVITY LEVEL	OPPORTUNITY COST	BOUND VALUE
1	U1	--	B	.0000010	--	INF
2	U2	--	B	.0107586	--	INF
3	U3	--	B	.0000010	--	INF
4	V1	--	B	.0032463	--	INF
5	V2	--	B	.0000010	--	INF
6	V3	--	B	.0074373	--	INF
7	V4	--	B	.0000010	--	INF
8	V5	--	B	.0000010	--	INF
9	V6	--	B	.0065950	--	INF
10	V7	--	B	.0000010	--	INF
11	V8	--	B	.0000010	--	INF
12	V9	--	B	.0000010	--	INF
13	V10	--	B	.0010643	--	INF
14	--SLACK	105	NB	--	5.4543824	INF
15	--SLACK	106	B	.0107576	--	INF
16	--SLACK	107	NB	--	7.2675860	INF
17	--SLACK	108	B	.0032453	--	INF
18	--SLACK	109	NB	--	8.1282583	INF
19	--SLACK	110	B	.0074363	--	INF
20	--SLACK	111	NB	--	4.3631157	INF
21	--SLACK	112	NB	--	1.2509771	INF
22	--SLACK	113	B	.0065940	--	INF
23	--SLACK	114	NB	--	.9662492	INF
24	--SLACK	115	NB	--	18.7373354	INF
25	--SLACK	116	NB	--	5.3922690	INF
26	--SLACK	117	B	.0010633	--	INF
27	--SLACK	D- 1	B	.1877854	--	INF
28	--SLACK	D- 2	B	.0937129	--	INF
29	--SLACK	D- 3	B	.1586697	--	INF
30	--SLACK	D- 4	B	.2960127	--	INF
31	--SLACK	D- 5	B	.2422727	--	INF
32	--SLACK	D- 6	B	.1854485	--	INF
33	--SLACK	D- 7	B	.1793784	--	INF
34	--SLACK	D- 8	B	.1982930	--	INF
35	--SLACK	D- 9	B	.1919999	--	INF
36	--SLACK	D- 10	B	.2322809	--	INF
37	--SLACK	D- 11	B	.5679378	--	INF
38	--SLACK	D- 12	NB	--	.0082485	INF
39	--SLACK	D- 13	NB	--	.0767510	INF
40	--SLACK	D- 14	B	.1348025	--	INF
41	--SLACK	D- 15	B	.0970580	--	INF
42	--SLACK	D- 16	B	.1650111	--	INF
43	--SLACK	D- 17	B	.1914567	--	INF
44	--SLACK	D- 18	B	.2773117	--	INF
45	--SLACK	D- 19	B	.2076822	--	INF
46	--SLACK	D- 20	B	.0072385	--	INF

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* PROBLEM NUMBER 1 *
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USING REGULAR
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SUMMARY OF RESULTS

VAR NO	VAR NAME	ROW NO	STATUS	ACTIVITY LEVEL	OPPORTUNITY COST	BOUND VALUE
47	--SLACK D-	21	B	.0349852	--	INF
48	--SLACK D-	22	B	.2767440	--	INF
49	--SLACK D-	23	B	.1616402	--	INF
50	--SLACK D-	24	B	.2762494	--	INF
51	--SLACK D-	25	B	.2724576	--	INF
52	--SLACK D-	26	B	.1959899	--	INF
53	--SLACK D-	27	B	.2590466	--	INF
54	--SLACK D-	28	B	.2460187	--	INF
55	--SLACK D-	29	B	.3210083	--	INF
56	--SLACK D-	30	B	.1213573	--	INF
57	--SLACK D-	31	B	.1645667	--	INF
58	--SLACK D-	32	B	.0474088	--	INF
59	--SLACK D-	33	B	.3367329	--	INF
60	--SLACK D-	34	B	.2030309	--	INF
61	--SLACK D-	35	B	.1339640	--	INF
62	--SLACK D-	36	B	.0347798	--	INF
63	--SLACK D-	37	B	.1965767	--	INF
64	--SLACK D-	38	B	.3210347	--	INF
65	--SLACK D-	39	B	.2293174	--	INF
66	--SLACK D-	40	B	.0295734	--	INF
67	--SLACK D-	41	B	.2135017	--	INF
68	--SLACK D-	42	B	.3257174	--	INF
69	--SLACK D-	43	B	.3607874	--	INF
70	--SLACK D-	44	B	.1346376	--	INF
71	--SLACK D-	45	B	.1190956	--	INF
72	--SLACK D-	46	B	.2537559	--	INF
73	--SLACK D-	47	B	.1951668	--	INF
74	--SLACK D-	48	B	.3404526	--	INF
75	--SLACK D-	49	B	.1045327	--	INF
76	--SLACK D-	50	B	.2621694	--	INF
77	--SLACK D-	51	B	.3383184	--	INF
78	--SLACK D-	52	B	.3084511	--	INF
79	--SLACK D-	53	B	.2109317	--	INF
80	--SLACK D-	54	B	.6408315	--	INF
81	--SLACK D-	55	B	.0355456	--	INF
82	--SLACK D-	56	B	.0130814	--	INF
83	--SLACK D-	57	B	.0356232	--	INF
84	--SLACK D-	58	B	.0143735	--	INF
85	--SLACK D-	59	B	.0622888	--	INF
86	--SLACK D-	60	B	.1318406	--	INF
87	--SLACK D-	61	B	.1170536	--	INF
88	--SLACK D-	62	B	.0901699	--	INF
89	--SLACK D-	63	B	.0049432	--	INF
90	--SLACK D-	64	B	.0218170	--	INF
91	--SLACK D-	65	B	.1762042	--	INF
92	--SLACK D-	66	B	.0759869	--	INF

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SUMMARY OF RESULTS

VAR NO	VAR NAME	ROW NO	STATUS	ACTIVITY LEVEL	OPPORTUNITY COST	BOUND VALUE
93	--SLACK	D- 67	B	.0805342	--	INF
94	--SLACK	D- 68	NB	--	.6018865	INF
95	--SLACK	D- 69	B	.0310154	--	INF
96	--SLACK	D- 70	B	.0484272	--	INF
97	--SLACK	D- 71	B	.0895409	--	INF
98	--SLACK	D- 72	B	.0372407	--	INF
99	--SLACK	D- 73	B	.0006540	--	INF
100	--SLACK	D- 74	B	.2173040	--	INF
101	--SLACK	D- 75	B	.1069407	--	INF
102	--SLACK	D- 76	B	.0831855	--	INF
103	--SLACK	D- 77	B	.1213998	--	INF
104	--SLACK	D- 78	B	.1552538	--	INF
105	--SLACK	D- 79	B	.0173918	--	INF
106	--SLACK	D- 80	B	.1980103	--	INF
107	--SLACK	D- 81	B	.0534289	--	INF
108	--SLACK	D- 82	B	.2295767	--	INF
109	--SLACK	D- 83	B	.0206094	--	INF
110	--SLACK	D- 84	B	.0104938	--	INF
111	--SLACK	D- 85	B	.0185319	--	INF
112	--SLACK	D- 86	B	.0919567	--	INF
113	--SLACK	D- 87	B	.0385137	--	INF
114	--SLACK	D- 88	B	.4229324	--	INF
115	--SLACK	D- 89	B	.0428767	--	INF
116	--SLACK	D- 90	B	.1324373	--	INF
117	--SLACK	D- 91	B	.0798809	--	INF
118	--SLACK	D- 92	B	.3027241	--	INF
119	--SLACK	D- 93	B	.0670942	--	INF
120	--SLACK	D- 94	B	.1839922	--	INF
121	--SLACK	D- 95	B	.0609909	--	INF
122	--SLACK	D- 96	B	.1464174	--	INF
123	--SLACK	D- 97	B	.3045598	--	INF
124	--SLACK	D- 98	NB	--	.1730669	INF
125	--SLACK	D- 99	B	.1945617	--	INF
126	--SLACK	D-100	B	.1697448	--	INF
127	--SLACK	D-101	B	.3734874	--	INF
128	--SLACK	D-102	B	.4541665	--	INF
129	--SLACK	D-103	B	.3679880	--	INF
130	--ARTIF	D-104	NB	--	.8407651	INF
131	--ARTIF	D-105	NB	--	-5.4543824	INF
132	--ARTIF	D-106	NB	--	0.0000000	INF
133	--ARTIF	D-107	NB	--	-7.2675860	INF
134	--ARTIF	D-108	NB	--	0.0000000	INF
135	--ARTIF	D-109	NB	--	-8.1282583	INF
136	--ARTIF	D-110	NB	--	0.0000000	INF
137	--ARTIF	D-111	NB	--	-4.3631157	INF
138	--ARTIF	D-112	NB	--	-1.2509771	INF

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SUMMARY OF RESULTS

VAR NO	VAR NAME	ROW NO	STATUS	ACTIVITY LEVEL	OPPORTUNITY COST	BOUND VALUE
139	--ARTIF	D-113	NB	--	0.0000000	INF
140	--ARTIF	D-114	NB	--	-.9662492	INF
141	--ARTIF	D-115	NB	--	-18.7373354	INF
142	--ARTIF	D-116	NB	--	-5.3922690	INF
143	--ARTIF	D-117	NB	--	0.0000000	INF

MAXIMUM VALUE OF THE OBJECTIVE FUNCTION = .840714

CALCULATION TIME WAS 1.3160 SECONDS FOR 32 ITERATIONS.

REFERENCES

1. Aigner, D. and S. F. Chu, "On Estimating the Industry Production Function," American Economic Review, LVIII (1968), 826-839.

2. Air Force Regulation 65:110. "Standard Aerospace Vehicle and Equipment Inventory Status and Utilization Reporting." Washington: Department of the Air Force (9 September 1977)(includes Change 1, 24 May 1978).

3. Ali, I., A Bessent, W. Bessent, and J. Kennington, "Data Envelopment Analysis of the Efficiency of Decision Making Units with the DEA3 Code (Version 3.0)," Research Report EPC 001, Educational Productivity Council, The University of Texas at Austin, (February 1982).

4. Banker, R., A. Charnes, W. W. Cooper, and A. Schinnar, "A Bi-Extremal Principle for Frontier Estimation and Efficiency Evaluations," Management Science, 27, 12 (1981), 1370-1382.

5. Barzily, Z., W. E. Caves, W. H. Marlow, A. Rao, S. Zacks, "Final Report on the Air Force Logistics Management Center Bibliography," (Capability and Readiness Assessment), Serial T-455/456, Institute for Management Science and Engineering, George Washington University (December 1981).

6. Bessent, A. and W. Bessent, "Determining the Comparative Efficiency of Schools Through Data Envelopment Analysis," Educational Administration Quarterly, 16, 2 (1980), 57-75.

7. Bessent, A., W. Bessent, A. Charnes, W. W. Cooper, and N. Thorogood, "Evaluation of Educational Program Proposals by Means of DEA," Research Report EPC 003, Educational Productivity Council, The University of Texas at Austin (February, 1982).

8. Bessent, A., W. Bessent, C. T. Clark, "Specification of Inputs and Outputs," Working Paper, Educational Productivity Council, The University of Texas at Austin (June 1982).

9. Bessent, A., W. Bessent, C. T. Clark, "Notes Comparing DEA to Statistical-Econometrics and Ratio Analysis," Working Paper, Educational Productivity Council, The University of Texas at Austin (August 1982).

10. Bessent, A., W. Bessent, J. Kennington, B. Reagan, "An Application of Mathematical Programming to Assess Managerial Efficiency in the Houston Independent School District," Management Science, 28, 12 (1982), 1355-1367.

11. Bowlin, William F., "Evaluating Budget Policies and Operations," Research Report EPC 009, Educational Productivity Council, The University of Texas at Austin (February 1979).

12. Campbell, A. K., "Efficient Government," Speech delivered at the University of West Florida, Pensacola, Florida (November 1979).

13. Charnes, A., and W. W. Cooper, "Programming with Linear Fractional Functionals," Naval Research Logistics Quarterly, 9, 3-4 (1962), 181-185.

14. Charnes, A., and W. W. Cooper, "Managerial Economics, Past, Present, and Future," Journal of Enterprise Management, 1, 1 (1978), pp. 5-23.

15. Charnes, A., and W. W. Cooper, "Auditing and Accounting for Program Efficiency in Not-For-Profit Entities," Accounting, Organization and Society, 5, 1 (1980), 87-107.

16. Charnes, A., and W. W. Cooper, "Management Science Relations for Evaluation and Management Accountability," Journal of Enterprise Management, 2, 2 (1980), 143-162.

17. Charnes, A., W. W. Cooper, A. Lewin, R. Morey, and J. Rousseau, "Efficiency Analysis with Non-Discretionary

Resources," Research Report CCS 379, Center for Cybernetic Studies, The University of Texas at Austin, Austin, Texas (1980).

18. Charnes, A., W. W. Cooper, and E. Rhodes, "Evaluating Program and Managerial Efficiency: An Application of Data Envelopment Analysis to Program Follow Through," Management Science, 27, 6 (1981), 668-697.

19. Charnes, A., W. W. Cooper, and E. Rhodes, "Measuring the Efficiency of Decision Making Units," European Journal of Operational Research, 2, 6 (1978), 429-444.

20. Clark, C. T., "Assessment of Logistics Capability," Unpublished paper (1981).

21. Clark, C. T., "An Investigation of Input-Output Measures for Evaluating the Efficiency of Air Force Tactical Fighter Wings," Research Report EPC 007, Educational Productivity Council, The University of Texas at Austin (December 1982).

22. Clark, C. T., "Evaluating the Efficiency of Air Force Combat Units," Research Report EPC 004, Educational Productivity Council, The University of Texas at Austin (March 1982).

23. Clark, C. T., "Locating the Nearest Facet," Unpublished experimental notes 8b and 13 (1983).

24. Culver, J. C., "A Report to the Senate Armed Services Committee," Washington: United States Senate (29 March 1977).

25. Definition of Air Force Integrated Readiness Measurement System (AFIRMS), Technical Proposal 8138/78, March 10, 1978, prepared by The Software Technology Company (Sof Tech), 460 Totten Pond Road, Waltham, MA 02154.

26. Elam, J., "Data Envelopment Analysis Code, Version 5 (BDEAV5)," documentation in progress, Educational Productivity Council, The University of Texas at Austin.

27. Farrell, M. J., "The Measurement of Productive Efficiency," Journal of the Royal Statistical Society, 120, 3 (1957), 253-290.

28. Forsund, F., C. Lovell, and P. Schmidt, "A Survey of Frontier Production Functions and of Their Relationship to Efficiency Measurement," Journal of Econometrics, 13, 1 (1980), pp. 5-25.

29. Frank, Steven A., W. B. Gruttske, W. H. Marlow, and S. J. Mathis, Jr., "Readiness Measurements via Subresource C-Ratings," The George Washington University Logistics Research Project for the Office of Naval Research, 25 November 1968.

30. Glover, F., J. Hultz, D. Klingman and J. Stutz, "Generalized Networks: A Fundamental Computer-Based Planning Tool," Management Science, 2, 12 (1978), 1209-1220.

31. Hanushek, E. A., "A Reader's Guide to Educational Production Functions," Paper prepared for the NIE National Invitation Conference of School Organization and Effects (December 1977).

32. Joint Chiefs of Staff Publication 6, "Joint Reporting Structure," (U)(JRS), Volume 11, Part 2, Chapter 1 (1977).

33. Jones, David C., General, "Readiness Year," TIG Brief, Number 7, Volume XXIII (9 April 1976), p. 2.

34. Keen, P. G. W., and M. S. Scott Morton, Decision Support Systems: An Organizational Perspective, Addison-Wesley Publishing Co., Reading Massachusetts (1978).

35. Lewin, A., and R. Morey, "Studies to Validate and Extend the Data Envelopment Analysis Method for Measuring the Relative Efficiency of Decision Making Units," Technical Report No. 1, Graduate School of Business Administration, Duke University (1980).

36. Lewin, A., R. Morey, and T. Cook, "Evaluating the Administrative Efficiency of Courts," Omega, 10, 4 (1982), pp. 401-411.

37. Mandell, J. H., "An Application of Data Envelopment Analysis to Corporations," Research Report EPC 010, Educational Productivity Council, The University of Texas at Austin (December 1982).

38. Mintzberg, H., D. Raisinghani, and A. Theore, "The Structure of 'Unstructured' Decision Processes," Administrative Science Quarterly (June 1976), pp. 246-275.

39. Musson, Thomas A., Lt. Col., USAF, "Readiness Measurement and Reporting Systems," Air War College Report No. 429 (April 1978), p. 25.

40. Nolte, Lawrence H., Jr., Lt. Col., USAF, Quotes obtained during discussions with Lt. Col. Nolte in February 1979. At that time he was project manager for developing the MAA User's Guide for Logistics.

41. Nolte, Lawrence H., Jr., Lt. Col., USAF, "Survey of Air Force Logistics Capability Assessment Concepts--Definitions--Techniques," Air Force Logistics Management Center Report 781029-1 (August 1980).

42. Reaves, L. J., "Using Data Envelopment Analysis to Operationalize the Concept of Equal Education Opportunity," unpublished doctoral dissertation (Austin, Texas: Department of Educational Administration, The University of Texas, 1983).

43. Rhodes, E. L., "Data Envelopment Analysis and Related Approaches for Measuring the Efficiency of Decision-Making Units with an Application to Program Follow Through in U.S. Education," unpublished doctoral dissertation (Pittsburgh, PA: School of Urban and Public Affairs, Carnegie-Mellon University, 1978).

44. Rogers, Albert G., Major General, USAF, Tactical Air Command Monthly Maintenance Summary, FY81 Wrap-Up, Langely, AFB, Virginia (1981).

45. Sherman, H. David, "Measurement of Hospital Efficiency Using Data Envelopment Analysis," unpublished DBA thesis (Cambridge, Mass., Graduate School of Business, Harvard University, 1981).

V I T A

Charles Terrance Clark was born on 29 December 1940 in New Orleans, Louisiana. He was graduated from high school in New Orleans in 1958. He attended Mississippi College, Louisiana State University, and then Pan American University in Texas. While at Pan American he was chosen for Who's Who Among Students in American Universities and Colleges in 1962 and 1963. In the spring of 1963 he received the Bachelor of Arts degree from Pan American University. In 1967 he received his commission in the USAF through the Officers Training School program. He served as an instructor in the Aircraft Maintenance Officer Course at Chanute Air Force Base, Illinois, until the spring of 1970. He attended the Air Force Institute of Technology where he received the degree of Master of Science in 1972. During the following four years he was assigned to the Logistics Systems Branch at Military Airlift Command Headquarters. In 1974 he became Chief of the Systems Branch and served in that capacity till 1976. He received the Air Force Meritorious Service Medal in 1976 for his achievements while assigned to Military Airlift Command. In the summer of 1976 he was transferred to Incirlik

Common Defense Installation in Adana, Turkey where he served first as Chief of the Field Maintenance Division and then as Chief of the Flightline Organizational Maintenance Division until reassignment to the United States in 1977. He was awarded the Air Force Meritorious Service Medal, First Oak Leaf Cluster for improvements he made in the maintenance operation during his tour in Turkey. In 1977 he began a tour of duty at the Air Force Logistics Management Center, Gunter Air Force Station, Alabama. While assigned to Gunter, he served as a project team member, then as a project manager and finally as the Chief of the Plans and Programs Division for the Center. After completion of the Gunter tour, he was awarded the Air Force Meritorious Service Medal, Second Oak Leaf Cluster in recognition of his achievements in establishing a comprehensive project management system. In January of 1981, he entered The University of Texas at Austin as a full time student under the sponsorship of the Air Force Institute of Technology.

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